LECTURE 1 - HOW TO DESIGN OPEN LOOP COMPARATORS

Topics:

- Comparator characterization
- Types of comparators
- Design of single-stage comparators
- Design of folded cascode comparators
- Design of two-stage comparators

What will I learn?

- How to design open loop comparators to satisfy a given set of specifications

Dr. Phillip Allen
Prof. Emeritus, ECE, Georgia Tech

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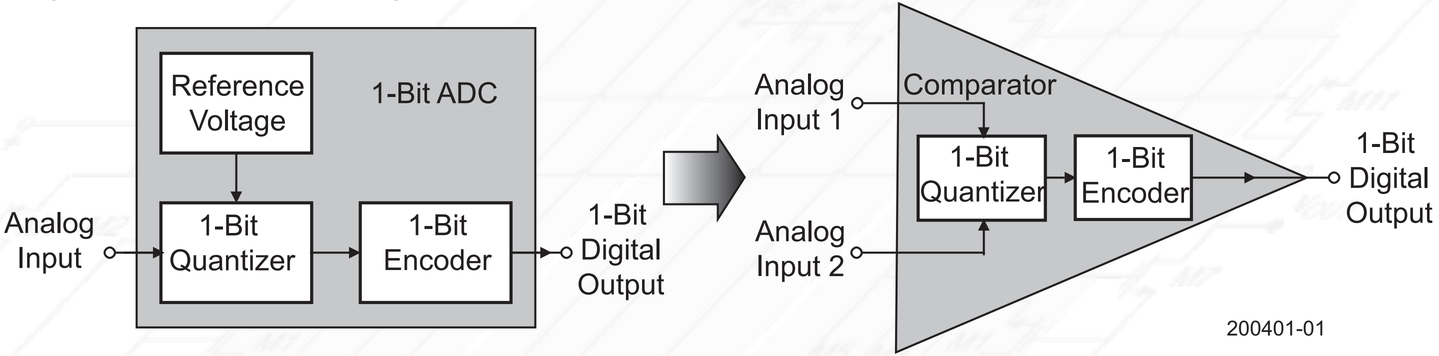
Characterization of Comparators

What is a Comparator?

- The comparator is a circuit that compares one analog signal with another analog signal or a reference voltage and outputs a binary signal based on the comparison
- The comparator is basically a 1-bit ADC



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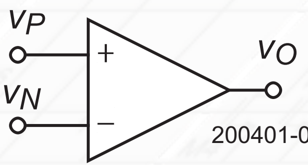
Comparator symbol:



$$v_O = \begin{cases} V_{OH} \text{ if } v_P > v_N \\ V_{OL} \text{ if } v_P < v_N \end{cases}$$

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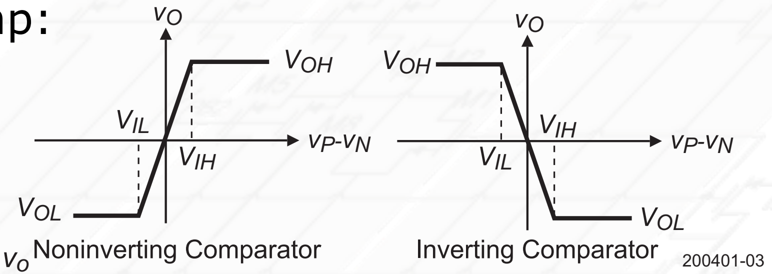
Characterization of Comparators

Characterization

The input to the comparator is analog and the characterization is essentially the same as an op amp:

- Common mode input range
- Input offset voltage
- Noise – transition uncertainty



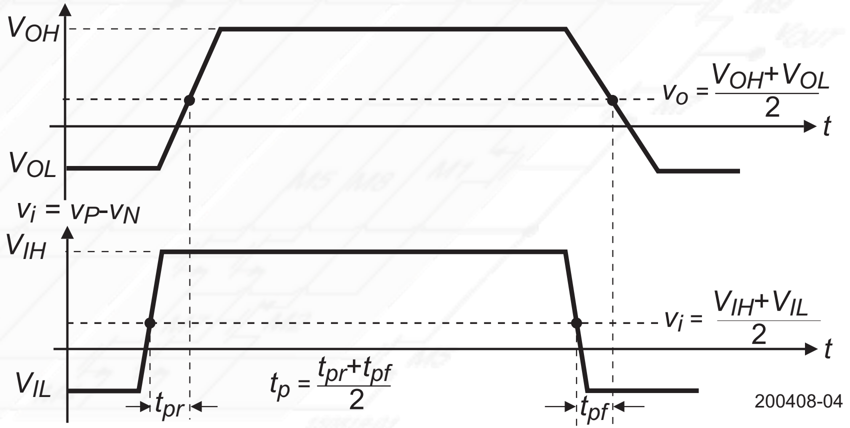
The transfer characterization:

$$|A_V| = \frac{V_{OH}-V_{OL}}{V_{IH}-V_{IL}}$$

Output characterization:

- V_{OH} and V_{OL}
- Slew rate
- Propagation delay time

$$t_p = \frac{t_{pr}+t_{pf}}{2}$$



Characterization of Comparators

Propagation delay time:

Assume the gain of the comparator is $A_v(s) = \dfrac{A_v(0)}{s/\omega_c+1} = \dfrac{A_v(0)}{s\tau_c+1}$

The step response is $v_o(t) = Av(0)[1 - e^{-t/\tau_c}]V_{in}$ where V_{in} is the magnitude of the step input.

The maximum slope occurs at t =0 and is $\dfrac{dv_o(t)}{dt}\Big|_{t=0} = \dfrac{A_v(0)}{\tau_c}V_{in}$

The rising propagation time, t_{pr}, is found as,

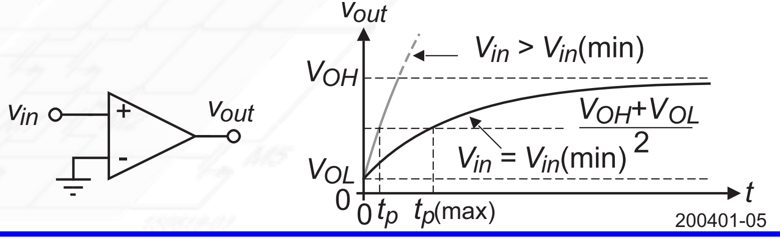
$$\frac{V_{OH}-V_{OL}}{2} = A_v(0)\left[1 - e^{-t_{pr}/\tau_c}\right]V_{in} \rightarrow\ t_{pr} = \tau_c ln\left[\frac{1}{1-\frac{V_{OH}-V_{OL}}{2A_v(0)V_{in}}}\right] = \tau_c ln\left[\frac{1}{1-\frac{V_{in}(min)}{2V_{in}}}\right]$$

Define k as the ratio of V_{in} to the minimum input voltage, V_{in}(min),

$$k = \frac{V_{in}}{V_{in}(min)} \quad \rightarrow \quad t_p = t_{pr} = \tau_c ln\left[\frac{2k}{2k-1}\right]$$

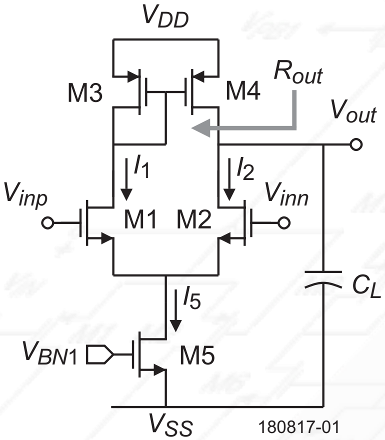
If k=1, t_p=0.693τ_c

If k=10, t_p=0.051τ_c

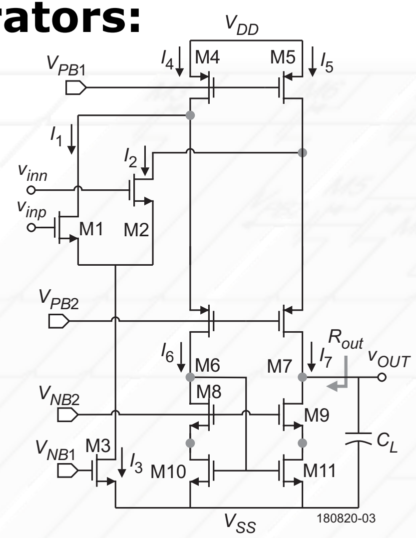


Open-Loop Comparators

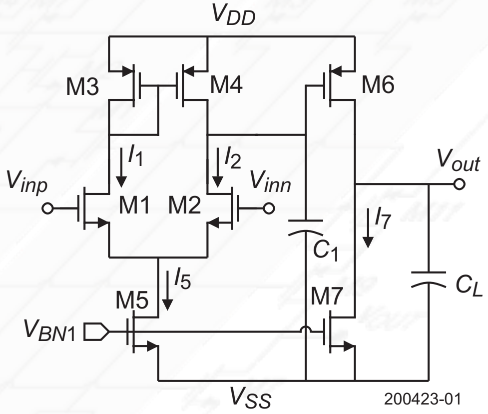
Possible open-loop comparators:



Design Procedure CO-1



Design Procedure CO-2



Design Procedure CO-3

Key Specifications:

- Minimum input voltage – $V_{in}(\text{min}) \Rightarrow A_v(0)$
- Propagation time delay - $t_p \Rightarrow GB$
- Slew Rate – $SR \Rightarrow I_{Sink/Source}$

Open-Loop Comparators

Slewing versus Linear Response

When does slew occur?

- Slewing occurs when the current in a capacitor becomes limited.

$$i = C\frac{dv}{dt} \rightarrow SR = \frac{dv}{dt} = \frac{i}{C}$$

- In a CMOS comparator, the input voltage must be large enough to cause the sinking/sourcing current to be on one or the other side of the differential amplifier input so that the current becomes constant.
- The value of V_{in} for this to occur is, $V_{in}(slew) \geq 2\sqrt{\frac{I_{Sink/Source}}{\frac{K'}{W/L}}}$
- This is generally much larger than the V_{in} for comparators and therefore most comparators will operate in the linear mode.
- If the comparator slews, the propagation delay time is given by,

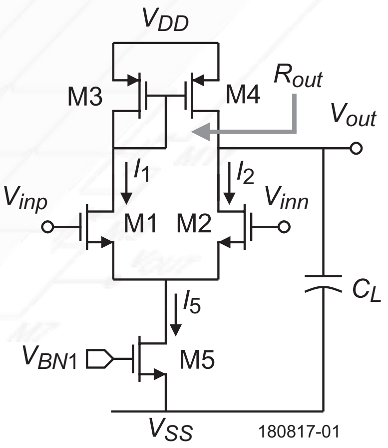
$$t_p = \Delta T = \frac{\Delta V}{SR} = \frac{V_{OH}-V_{OL}}{2\cdot SR}$$

where SR is the slew rate of the comparator.

Simple One-Stage Comparator Design

Design Procedure – CO1:

Step	Design Equations	Comments
This design procedure assumes $V_{in} = V_{in}(\text{min})$		
1	$A_v(0)=(V_{OH}-V_{OL})/V_{in}(\text{min})$	V_{OH}, V_{OL}, $V_{in}(\text{min})$ given
2	$\frac{g_{m1}}{C_L} = GB = \frac{A_v(0)}{\tau_c} = \frac{0.693A_v(0)}{t_p} \Rightarrow g_{m1} = \frac{0.693A_v(0)}{t_p}C_L$	Assumes t_p and C_L are given.
3	$\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{g_{m1}{}^2}{K_N' I_5}$	Choose I_5
4	$\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{I_5}{K'_3[V_{DD} - ICMR^+ - \|V_{T3}\| + V_{T1}]^2}$	$ICMR^+ = V_{in}(\text{max})$ specified, M3 = M4
5	$\frac{W_5}{L_5} = \frac{2I_5}{K_N'(ICMR^- - V_{GS1})^2}$, $V_{GS1} = \sqrt{I_5/K_N'\left(\frac{W_1}{L_1}\right)} + V_{T1}$	Note that $ICMR^-$ must be greater than V_{GS1}
6	Check power dissipation, $P_{diss} = V_{DD}I_5$	Change I_5 in step 3 to meet power dissipation



Simple One-Stage Comparator Design

Example 1

If $K_N' = 120\mu A/V^2$, $K_P' = 25\mu A/V^2$, $V_{TN} = |V_{TP}| = 0.5V$, $\lambda_N = 0.06V^{-1}$, and $\lambda_P = 0.08V^{-1}$, use Design Procedure CO1 to design a single-pole CMOS comparator that meets the following specifications. Assume $C_L = 10pF$.

$t_p = 10\ \mu s$	$V_{DD} = 2.5V$	$V_{OH} = 2V$	$V_{OL} = 0.5V$	$V_{in}(\min) = 10mV$	$1V \le ICMR \le 2V$

1.) $A_v(0)=(V_{OH}-V_{OL})/V_{in}(\min) = 1.5V/10mv = 150\ V/V$

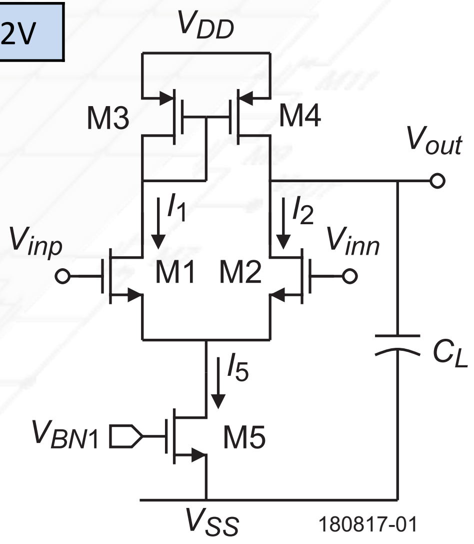
2.) $g_{m1} = \frac{0.693A_v(0)}{t_p}C_L = \frac{0.693\times150\times10pF}{10\mu s} = 104\mu S$

3.) Choose $I_5 = 10\mu A$. $\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{{g_{m1}}^2}{2K_N' I_3} = \frac{(104\times10^{-6})^2}{120\times10^{-6}\times10\times10^{-6}} = 9$

4.) $\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{I_5}{K'_3[V_{DD}-ICMR^{+}-|V_{T3}|+V_{T1}]^2} = \frac{10\times10^{-6}}{25\times10^{-6}[2.5-2]^2} = 1.6 \rightarrow\ 2$

5.) $V_{GS1} = \sqrt{I_5/K_N'\left(\frac{W_1}{L_1}\right)} + V_{T1} = \sqrt{10\times10^{-6}/(120\times10^{-6}\times9)} + 0.5 = 0.596V$,

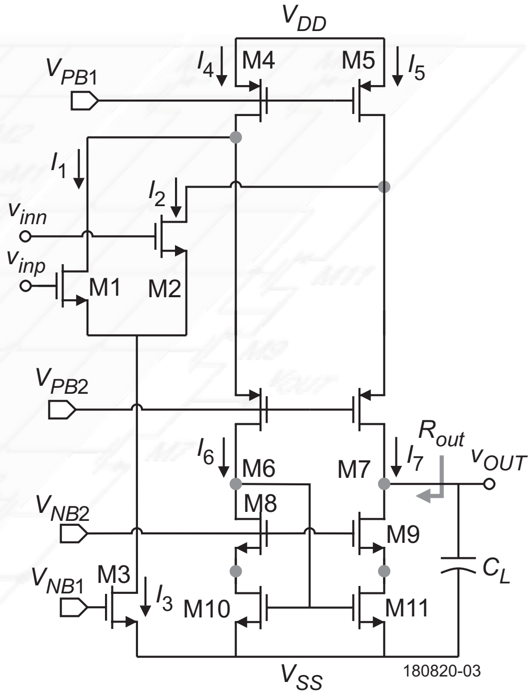
$\frac{W_5}{L_5} = \frac{2I_5}{K_N'(ICMR^{-}-V_{GS1})^2} = \frac{20}{120(1-0.596)^2} = 1$



Folded Cascode Comparator Design

Design Procedure – CO2

Step	Design Equations	Comments
	This design procedure assumes $V_{in} = V_{in}(\min)$	
1	$A_v(0)=(V_{OH}-V_{OL})/V_{in}(\min)$	V_{OH}, V_{OL}, $V_{in}(\min)$ given
2	$g_{m1} = \dfrac{0.693A_v(0)}{t_p}C_L$	Assumes t_p and C_L are given.
3	$\dfrac{W_1}{L_1} = \dfrac{W_2}{L_2} = \dfrac{{g_{m1}}^2}{2K'_N I_3}$	Choose I_3 and I_4 and I_5
4	$\dfrac{W_3}{L_3} = \dfrac{2I_3}{K'_N(ICMR^- - V_{GS1})^2}, V_{GS1} = \sqrt{\dfrac{I_3}{K'_N(\frac{W_1}{L_1})}} + V_{T1}$	Note that $ICMR^-$ must be greater than V_{GS1}
5	$W_4/L_4 = W_5/L_5 = 4I_4/[K'_P(V_{DD}-V_{OH})^2]$	$V_{SD4}(\text{sat})=0.5[V_{DD}-V_{OH}]$
6	$W_6/L_6 = W_7/L_7 = 4I_6/[K'_P(VDD-V_{OH})^2]$	$V_{SD6}(\text{sat})=0.5[V_{DD}-V_{OH}]$
7	$\dfrac{W_8}{L_8} = \dfrac{W_9}{L_9} = \dfrac{W_{10}}{L_{10}} = \dfrac{W_{11}}{L_{11}} = \dfrac{4I_6}{K'_N {V_{OL}}^2}$	$V_{DS8}(\text{sat})=0.5[V_{OL}-V_{SS}]$
8	$P_{diss} = V_{DD}(I_4+I_5)$	Can change currents if too large



Folded Cascode Comparator Design

Example 2

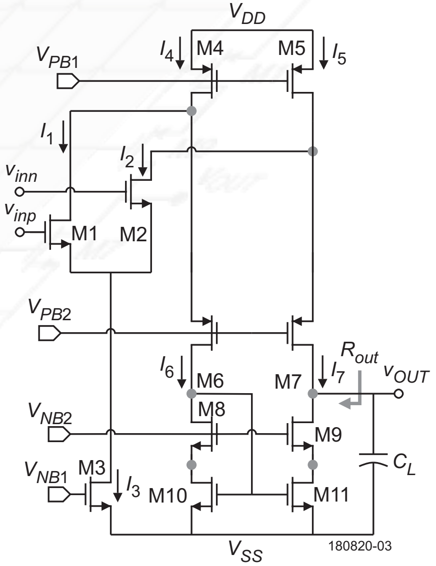
If $K_N' = 120\mu A/V^2$, $K_P' = 25\mu A/V^2$, $V_{TN} = |V_{TP}| = 0.5V$, $\lambda_N = 0.06V^{-1}$, and $\lambda_P = 0.08V^{-1}$, design a folded cascode comparator that meets the following specifications. Assume

V_{DD} = 2.5V	t_p = 10 μs	V_{OH} = 2.25V	V_{OL} = 0.25V	V_{in}(min) = 1mV	1V ≤ $ICMR$ ≤ 2V

1. $A_v(0)=(V_{OH}-V_{OL})/V_{in}(\text{min}) = 1.75V/1mV = 1750V/V$
2. $g_{m1} = \frac{0.693A_v(0)}{t_p}C_L = \frac{0.693\times1750\times10pF}{10\mu s} = 1.213\text{mS}$
3. Choose I_3 = 100μA. $\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{g_{m1}{}^2}{2K_N' I_3} = \frac{(1.218\times10^{-3})^2}{2\times120\times10^{-6}\times100\times10^{-6}} = 61.3$
4. $V_{GS1} = \sqrt{I_3/K_N'\left(\frac{W_1}{L_1}\right)} + V_{T1} = 0.616V \Rightarrow$

$$\frac{W_3}{L_3} = \frac{2I_3}{K_N'(ICMR^- - V_{GS1})^2} = \frac{200}{120(1-0.61)^2} = 11.3$$

5. Choose $I_4 = I_5$ 125μA. $W_4/L_4 = W_5/L_5 = 4I_4/[K'_P(V_{DD}-V_{OH})^2] = 500/[25(0.25)^2] = 320$
6. $W_6/L_6 = W_7/L_7 = 4I_6/[K'_P(VDD-V_{OH})^2] = 300/[25(0.25)^2] = 192$
7. $\frac{W_8}{L_8} = \frac{W_9}{L_9} = \frac{W_{10}}{L_{10}} = \frac{W_{11}}{L_{11}} = \frac{4I_6}{K_N' V_{OL}{}^2}$ = 300/[120(0.25)²] = 40



Two-Stage Comparators

Propagation delay time:

The step response of a comparator with two real axis poles ($p_1 \neq p_2$) is,

$$v_{out}(t) = A_V(0)V_{in}\left[1 + \frac{p_2 e^{-t|p_1|}}{p_1 - p_2} - \frac{p_1 e^{-t|p_2|}}{p_1 - p_2}\right]$$

Normalizing in amplitude and time gives,

$$v'_{out}(t_n) = \frac{v_{out}(t)}{A_v(0)V_{in}} = 1 + \frac{m}{1-m}e^{-t_n} - \frac{1}{1-m}e^{-mt_n}$$

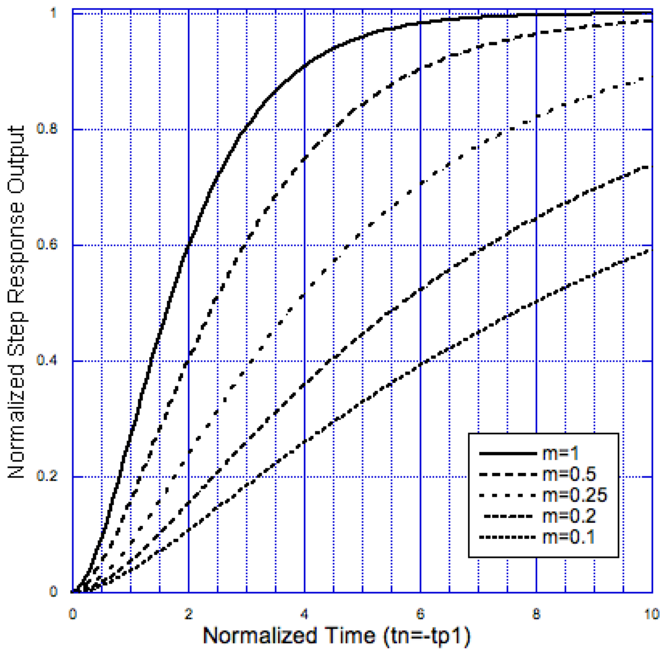
where $m = p_2/p_1 \neq 1$ and $t_n = t|p_1|$

If $p_1 = p_2$, then the normalized output voltage is,

$$v'_{out}(t) = 1 - e^{-t|p_1|} - t|p_1|e^{-t|p_1|}$$

$$v'_{out}(t_n) = 1 - e^{-t_n} - t_n e^{-t_n}$$

A graph of the output is shown.



Two-Stage Comparators

Propagation delay time - Continued:

To find the propagation delay time, t_p, equate amplitude unnormalized output to $0.5(V_{OH}-V_{OL})$.

$$v_{out}(t_{pn}) = A_v(0)V_{in}\left[1 - \frac{m}{m-1}e^{-t_{pn}} + \frac{1}{m-1}e^{-mt_{pn}}\right] = 0.5(V_{OH} - V_{OL})$$

Approximating the exponential terms with a power series and taking the dominant terms, the propagation time delay can be expressed as,

$$t_{pn} \approx \sqrt{\frac{V_{OH}-V_{OL}}{mA_v(0)V_{in}}} = \sqrt{\frac{V_{in}(min)}{mV_{in}}} = \frac{1}{\sqrt{mk}} \Rightarrow t_p = \frac{1}{|p_1|\sqrt{mk}} = \frac{1}{\sqrt{kp_1p_2}}$$

If we assume k=1 where V_{in} = V_{in}(min), then

$$|p_1| = \frac{1}{|p_2|t_p^2} \quad \text{and} \quad |p_1| = \frac{1}{R_1C_1} = \frac{(\lambda_n+\lambda_p)I_5}{2C_1} \quad \text{also } |p_2| = \frac{1}{R_2C_L} = \frac{(\lambda_n+\lambda_p)I_7}{C_L}$$

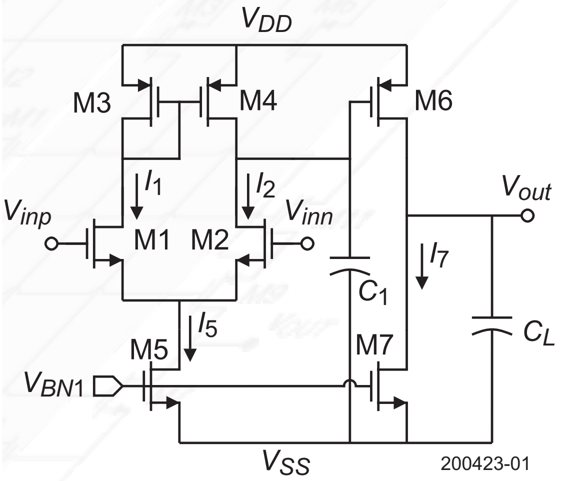
where C_1 is the capacitance to ground at the output of the first stage. Equating the two expressions for $|p_1|$, substituting for $|p_2|$, and solving for C_1 gives,

$$C_1 = \frac{(\lambda_n+\lambda_p)^2 I_5 I_7 t_p^2}{2C_L}$$

Two-Stage Comparator Design

Design Procedure – CO3:

Step	Design Equations	Comments
	This design procedure assumes V_{in} = V_{in}(min) or k = 1	
1	$A_v(0)=(V_{OH}-V_{OL})/V_{in}(\text{min})$	V_{OH}, V_{OL}, V_{in}(min) given
2	$C_1 = \dfrac{(\lambda_n+\lambda_p)^2 I_5 I_7 t_p^2}{2C_L}$	Choose I_5 and I_7. t_p and C_L are assumed to be given.
3	$\dfrac{W_6}{L_6} = \dfrac{2I_7}{K'_p(V_{DD}-V_{OH})^2}$ and $\dfrac{W_7}{L_7} = \dfrac{2I_7}{K'_n(V_{OL}-V_{SS})^2}$	V_{DD}-V_{OH} = V_{sd6}(sat) and V_{OL} = V_{ds7}(sat)
4	$\dfrac{W_1}{L_1} = \dfrac{W_2}{L_2} = \dfrac{A_v^2(0) I_5 I_7 (\lambda_n+\lambda_p)^2}{4K'_n K'_p (W_6/L_6)}$	$g_{m1} = \dfrac{A_v(0)}{g_{m6}R_1R_2}$ $g_{m1} = \sqrt{I_5 K'_n (W_1/L_1)}$
5	$\dfrac{W_5}{L_5} = \dfrac{I_5}{I_7}\dfrac{W_7}{L_7}$ or $\dfrac{W_5}{L_5} = \dfrac{2I_5}{K'_N(ICMR^- - V_{GS1})^2}$, $ICMR^- > V_{GS1}$	W_5/L_5 should satisfy both relationships if possible
6	$\dfrac{W_3}{L_3} = \dfrac{W_4}{L_4} = \dfrac{I_5}{K'_3[V_{DD} - ICMR^+ - \lvert V_{T3}\rvert + V_{T1}]^2}$	$ICMR^+ = V_{in}$(max) specified, M3 = M4



Two-Stage Comparator Design

Example 3

If $K_N' = 120\mu A/V^2$, $K_P' = 25\mu A/V^2$, $V_{TN} = |V_{TP}| = 0.5V$, $\lambda_N = 0.06V^{-1}$, and $\lambda_P = 0.08V^{-1}$, design a two-stage comparator that meets the specifications. Assume $V_{in} = V_{in}(\text{min})$ and $C_L = 10pF$.

$V_{DD} = 2.5V$	$t_p = 1\ \mu s$	$V_{OH} = 2.25V$	$V_{OL} = 0.25V$	$V_{in}(\text{min}) = 1mV$	$1V \le ICMR \le 2V$

1. $A_v(0)=(V_{OH}-V_{OL})/V_{in}(\text{min}) = 2V/1mV = 2000V/V$
2. Choose $I_5=10\mu A$ and $I_7=50\mu A$

$$C_1 = \frac{(\lambda_n+\lambda_p)^2 I_5 I_7 t_p^2}{2C_L} = \frac{(0.06+0.08)^2 10\mu A\times 50\mu A\times 1\mu s^2}{2\times 10pF} = 0.49pF$$

3. $\frac{W_6}{L_6} = \frac{2I_7}{K_p'(V_{DD}-V_{OH})^2} = \frac{2\times 50\mu A}{25\times 10^{-6}\times 0.25^2} = 64$ and $\frac{W_7}{L_7} = \frac{2I_7}{K_n'(V_{OL}-V_{SS})^2} = \frac{2\times 50\mu A}{120\times 10^{-6}\times 0.25^2} = 13.33$

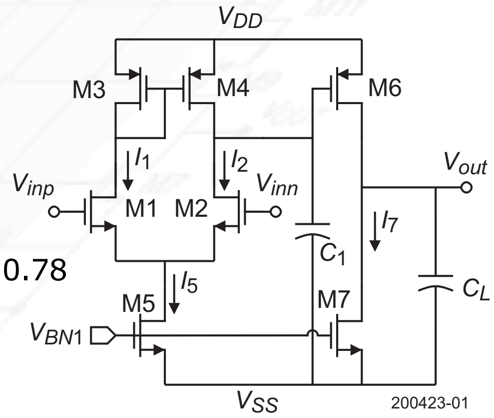
4. $\frac{W_1}{L_1} = \frac{W_2}{L_2} = \frac{A_v^2(0) I_5 I_7 (\lambda_n+\lambda_p)^2}{4K_n' K_p' (W_6/L_6)} = \frac{2000^2\times 10\mu A\times 50\mu A\times (0.06+0.08)^2}{4\times 120\times 10^{-6}\times 25\times 10^2\times 64} = 51.04$

5. $\frac{W_5}{L_5} = \frac{I_5}{I_7}\frac{W_7}{L_7} = \frac{10}{50}\times 13.33 = 2.67$ or $\frac{W_5}{L_5} = \frac{I_5}{K_N'(ICMR^- - V_{GS1})^2} = \frac{20}{120(1-0.54)^2} = 0.78$

$$V_{GS1} = \sqrt{I_5/K_N'\left(\frac{W_1}{L_1}\right)} + V_{T1} = 0.54V$$

6. $\frac{W_3}{L_3} = \frac{W_4}{L_4} = \frac{I_5}{K'_3[V_{DD} - ICMR^+ - |V_{T3}| + V_{T1}]^2} = \frac{10}{25\times(2.5-2)^2} = 1.6$

It turns out that $C_{GS6} = 52.5fF$ so C_1 should be changed to 0.438pF. Also, $m=[I_5/(2xI_7)](C_1/C_L) = 0.49$.



Open Loop Comparator Design

Summary:

- Types of op amps
- Simple one-stage comparator
  - Design procedure CO1
  - Example 1
- Folded cascode comparator
  - Design procedure CO2
  - Example 2
- Two-stage comparator
  - Design procedure CO3
  - Example 3

Change Tracking

Date	Slide No.	Correction or change
11/11/19	7	Steps 1 and 8 have been changed to include the influence of I_{10}
11/11/19	9	Step 7 has been modified to include I_{10}
6/12/20	6	Added to the second bullet – "so that the current becomes constant."
6/12/20	9	Added under comments on line 8 "Can change currents if too large."