FEEDBACK - PART 3

Topics in Part 3

- Feedback stability
  - Routh-Hurwitz criterion
  - Open-loop frequency response
  - Nyquist stability criterion
- Stability simulation methods
- Brute force simulation methods
- Unusual circuit behavior achieved by feedback
- Summary

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Feedback Stability

Methods

Stable or Not?

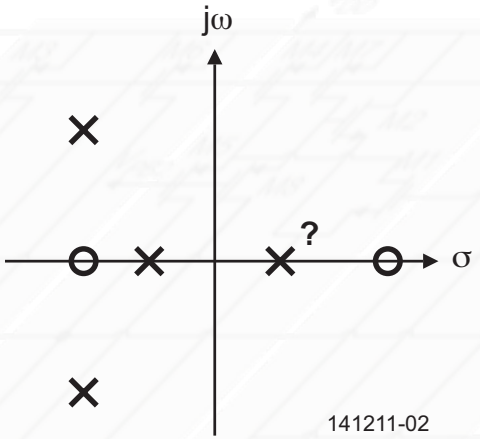
Will the system oscillate?

- Routh-Hurwitz criterion
- Open-loop Frequency Response
- Nyquist's criterion

Degree of Stability

How close to oscillation is the system?

- Gain and phase margin
- -1 encirclement



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Feedback Stability

Routh-Hurwitz Criterion†

Examines the roots of a characteristic polynomial, $P(s)$.

$$P(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n = 0$$

Form the array,

s^n	a_0	a_2	a_4	a_6	$\cdots$
s^{n-1}	a_1	a_3	a_5	a_7	$\cdots$
s^{n-2}	b_1	b_2	b_3	$\cdots$	
s^{n-3}	c_1	c_2	$\cdots$		
s^{n-4}	d_1	$\cdots$			
$\vdots$	$\vdots$				

where

$$b_i = \frac{a_1 a_{2i} - a_0 a_{2i+1}}{a_1}$$

$$c_i = \frac{b_1 a_{2i+1} - a_1 b_{i+1}}{b_1}$$

$$d_i = \frac{c_1 b_{i+1} - b_1 a_{2i+1}}{c_1}$$

$$\vdots$$

The number of sign changes in the left-hand column of the array is equal to the number of roots with positive-real parts.

† E.J. Routh, " A Treatise on the Stability of a Given State of Motion," London: Macmillan, 1877.

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Feedback Stability

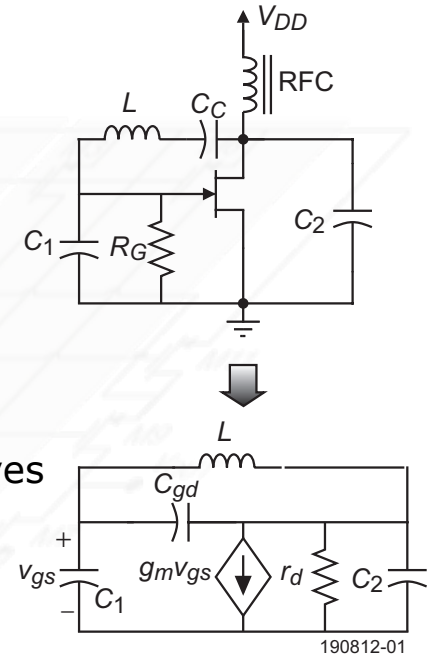
Colpitts Oscillator

Use the Routh-Hurwitz criterion to find the frequency of oscillation.

The denominator can be found as,

$$D(s) = \frac{L(C_1C_2 + C_1C_{gd} + C_2C_{gd})s^3 + L(C_1g_d + C_{gd}g_d + g_mC_{gd})s^2 + (C_1 + C_2)s + (g_m + g_d)}{sL}$$

$$\approx \frac{LC_1C_2s^3 + (LC_1g_d)s^2 + (C_1 + C_2)s + (g_m + g_d)}{sL}$$



Assuming that C_1 = 750pF, C_2 = 2500pf, L=40μH, and r_d = 50kΩ, gives

$$D(s) \approx 75p^3 + 0.006p^2 + 0.325p + 2\text{x}10^{-5} + g_m = 0$$

where $p = s/10^8$. Applying the Routh-Hurwitz criterion gives,

p^3	75	0.325
p^2	0.006	$2\times10^{-5} + g_m$
p^1	$0.075 - 12.5\times10^3 g_m$	0
p^0	$2\times10^{-5} + g_m$	0

For all elements of the array to be positive requires that,

$$g_m > \frac{0.075}{12.5\times10^3} = 6\mu S$$

Therefore, the oscillation frequency = 1.048MHz

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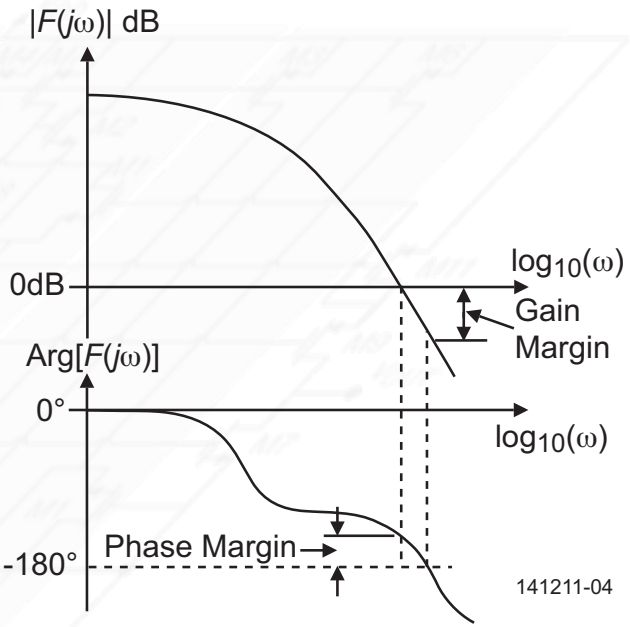
Feedback Stability

Open-Loop Frequency Response

- Gain Margin – The gain margin is the magnitude in dB of the return ratio below the 0-dB level at the frequency when the phase angle of the return ratio equals -180°.

- Phase Margin – The phase margin is the phase difference in degrees of the return ratio above -180° at the frequency when the magnitude of the return ratio is unity or 0dB.



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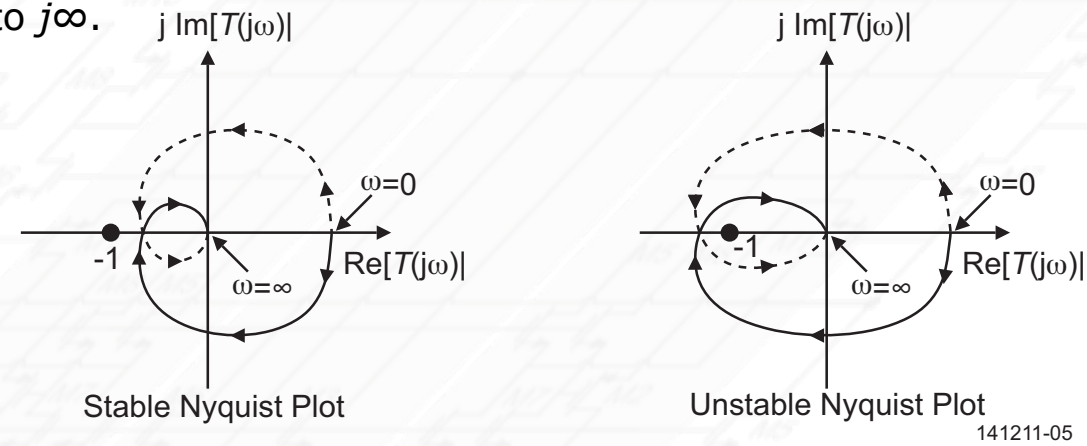
Feedback Stability

Nyquist Stability Criterion†

The Nyquist stability criterion is based on the return difference, $F(s)$. However, it is more convenient to use the return ratio, $T(s)$.

A necessary and sufficient condition for a feedback amplifier to be stable is that the Nyquist plot of the return ratio should not encircle the -1+j0 in the $T(s)$ plane.

The Nyquist plot is achieved by plotting the magnitude and phase of $T(s)$ as a function of s varying from 0 to $j\infty$.



† H. Nyquist, "Regeneration Theory," *Bell System Technical J.,* vol. 11, no.1, pp. 126-147, 1932.

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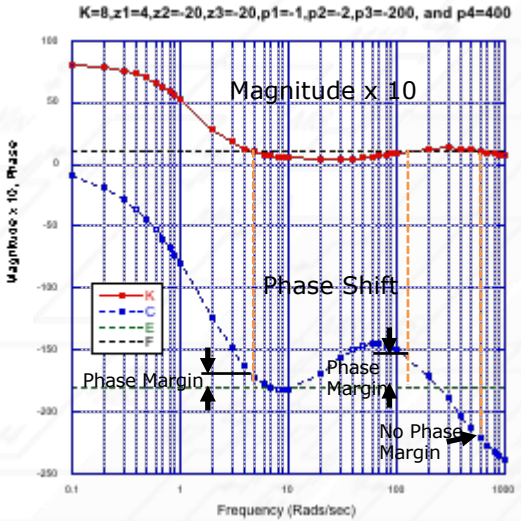
Nyquist Stability Analysis

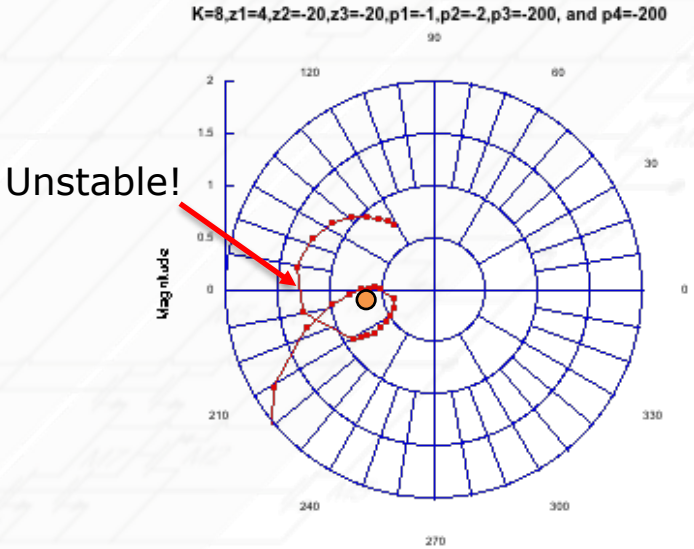
Example

Consider the return difference:

$$T(s) = \frac{8\left(\frac{s}{4}-1\right)\left(\frac{s}{20}+1\right)\left(\frac{s}{20}+1\right)}{\left(\frac{s}{1}+1\right)\left(\frac{s}{2}+1\right)\left(\frac{s}{200}+1\right)\left(\frac{s}{400}+1\right)}$$

The frequency response and Nyquist plot are shown below:





If the loop magnitude ≥ 1 at 180° the system is unstable!

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Stability Simulation Methods

Overview

- Problem is that the return ratio requires the loop to be open and dc conditions are not preserved
  - Keep the loop closed – transient response
  - Open the loop and maintain dc conditions
- Simulation methods
  - Middlebrook's Null Double Injection
    - When the loop is cut, there is a voltage, v_f and a current i_f associated with the break.
    - This method applies a voltage, v_t, and a current, i_t, that causes v_f and i_f to become zero.
    - The voltage and current return ratio can be found from these results.
    - This method assumes the forward gain is unilateral.
  - Two-port method using ABCD parameters
  - Low and high pass filters
  - STB
  - ResBias
    - A resistor with one value for AC and another for DC

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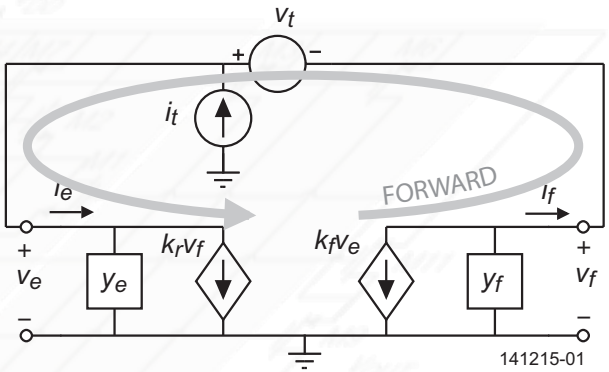
Two-Port Method†

Feedback parameters (*a, f, A,* and *T*) are computed using two-port parameter techniques*

- If v_t and i_t are zero, then the normal return ratio and the reverse return ratio are:

$$\left.\begin{aligned} T_{Forward} &= -\frac{v_e}{v_e'}(k_r = 0) = \frac{k_f}{y_e + y_f} \\ T_{Reverse} &= -\frac{v_f}{v_f'}(k_f = 0) = \frac{k_r}{y_e + y_f} \end{aligned}\right\} \quad T = T_{Forward} + T_{Reverse} = \frac{k_v + k_f}{y_e + y_f}$$



- The *ABCD* parameters can be written as:

$$\begin{aligned} i_f &= Ai_t + Bv_t \\ v_e &= Ci_t + Dv_t \end{aligned}$$

where, $A = i_f\Big|_{i_t = 1, v_t = 0}$, $B = i_f\Big|_{i_t = 0, v_t = 1}$

$C = v_e\Big|_{i_t = 1, v_t = 0}$, $D = v_e\Big|_{i_t = 0, v_t = 1}$

† M. Tian, V. Visvanathan, J. Hantgan, and K. Kundert, "Striving for Small-Signal Stability," *IEEE Circuits & Devices*, Jan. 2001, pp. 31-41.
* This analysis assumes that the feedback network is linear and time-invariant

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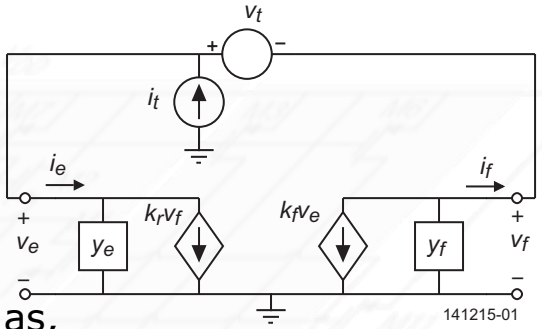
Two-Port Method - Continued

- The circuit equations at the input side and the output side of the feedback loop are:

$$\sum \text{Currents at input} = -y_e v_e + i_t + i_f - k_r(v_e - v_t) = 0$$

$$\sum \text{Currents at output} = k_f v_e + i_f + y_f(v_e - v_t) = 0$$



- These equations can be solved for the *ABCD* parameters as,

$$A = \frac{-(k_f + y_f)}{k_f + k_r + y_e + y_f}, \quad B = \frac{-(y_e y_f - k_f k_r)}{k_f + k_r + y_e + y_f}$$

$$C = \frac{1}{k_f + k_r + y_e + y_f}, \quad D = \frac{k_r + y_f}{k_f + k_r + y_e + y_f}$$

- Solving these four equations for k_f, k_r, y_e, and y_f, gives,

$$k_f = \frac{AD - BC - A}{C}, \qquad k_r = \frac{AD - BC + D}{C}$$

$$y_e = \frac{1 - AD + BC + A - D}{C}, \quad y_f = \frac{BC - AD}{C}$$

Two-Port Method - Continued

So how does STB work in Spectre?

1. Computes *ABCD* parameters by measuring *A* (i_f with i_t=1 and v_t=0), *B* (i_f with i_t=0 and v_t=1), *C* (v_e with i_t=1 and v_t=0), and *D* (v_e with i_t=0 and v_t=1).
2. Uses the *ABCD* parameters to find k_f, k_r, y_e, and y_f and from these parameters finds $T_{Forward}$, $T_{Reverse}$, and *T*.

$$T = \frac{2(AD - BC) - A + D}{2(BC - AD) + A - D + 1}$$

$$T_{\text{forward}} = \frac{AD - BC - A}{2(BC - AD) + A - D + 1}$$

$$T_{\text{reverse}} = \frac{AD - BC + D}{2(BC - AD) + A - D + 1}$$

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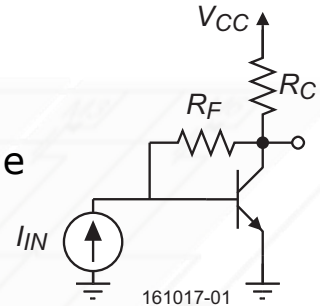
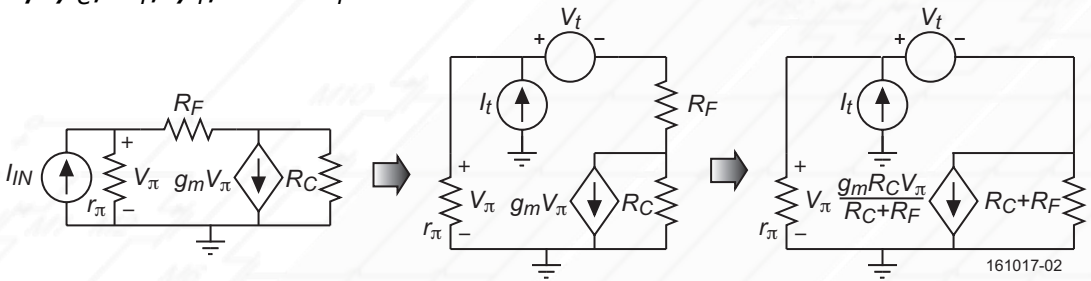
Two-Port Method - Continued

Example:

Find the forward loop gain of the feedback circuit shown.

While one could find the ABCD parameters, it is easier in this example to identify y_e, k_f, y_f, and k_r as follows.





Therefore,

$$y_e = g_\pi,\ k_f = \frac{g_m R_C}{R_C + R_F},\ y_f = \frac{1}{R_C + R_F},\ \text{and } k_r = 0$$

The forward loop gain is,

$$T_{Forward} = \frac{k_f}{y_e + y_f} = \frac{g_m r_\pi R_C}{r_\pi + R_C + R_F} = \frac{\beta R_C}{r_\pi + R_C + R_F}$$

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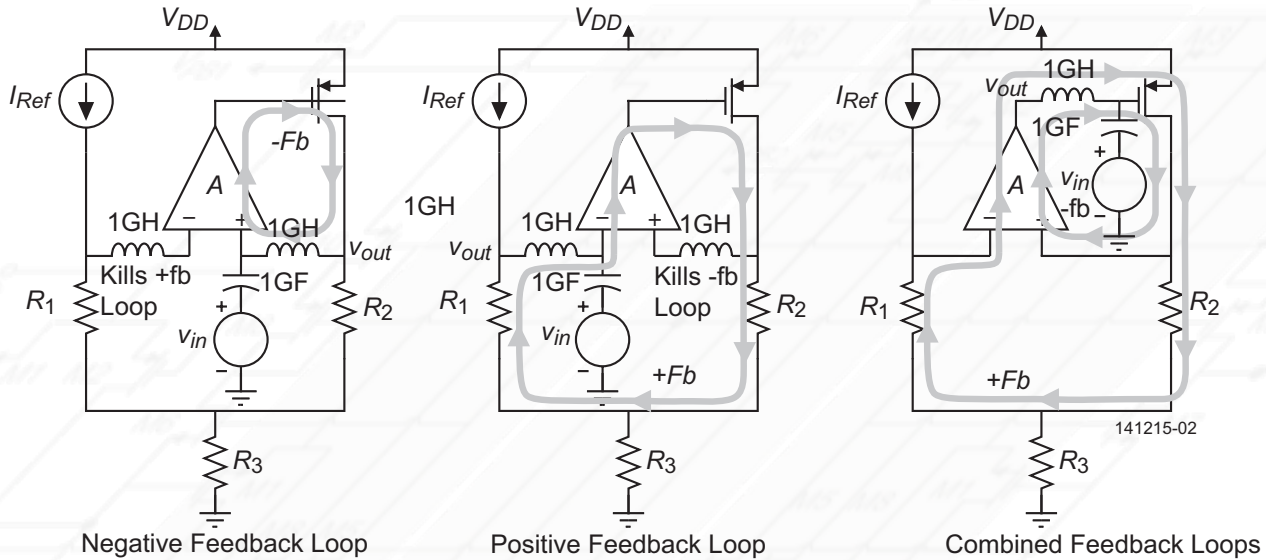
Low and High Pass Filter Methods

Insertion of Large *L* and Large *C* to measure return ratio:



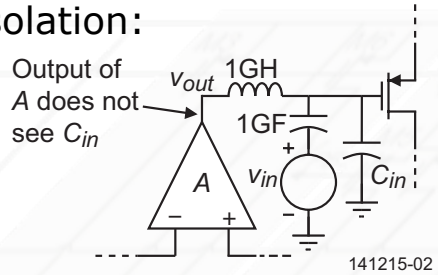
Potential Problems:

- Large inductors and large capacitors causes numerical ill-conditioning in the simulator
- The large *L* can "shield" capacitive parasitics (next slide)

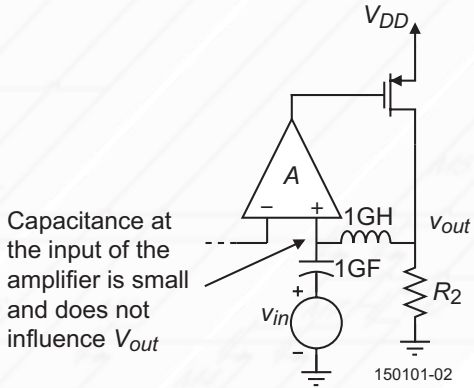
Low-Pass Filter Methods

Advice

Potential error using inductive isolation:



Use the inductor where its influence on parasitic capacitors will be minimized.



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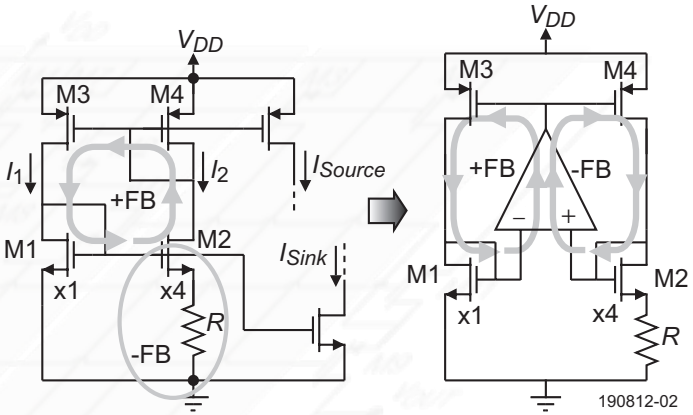
Feedback and Unusual Circuit Behavior

Application of Feedback to Achieve Desired Performance

It has been observed that unusual circuit behavior is accompanied with both positive and negative feedback.

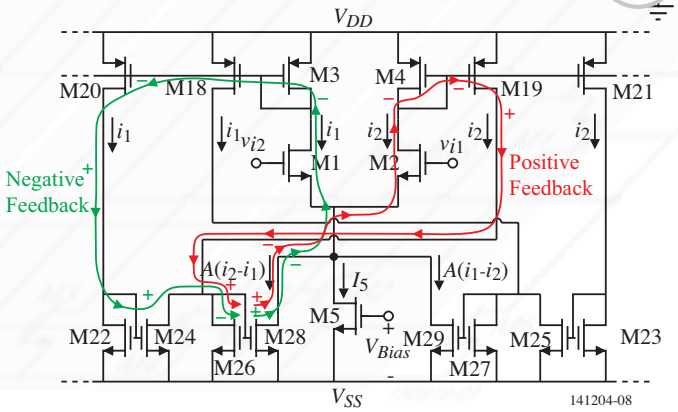
- Power supply independent biasing (also PTAT circuits)



- Current boosting

$$i_{out} = \frac{bI_5\left[\exp\left(\frac{v_{in}}{nV_t}\right)-1\right]}{1+A-(A-1)\exp\left(\frac{v_{in}}{nV_t}\right)}$$

where $A = \frac{W_{28}/L_{28}}{W_{26}/L_{26}}$



Summary

Feedback Applied to Analog IC Design

- Types of feedback
- Definitions
- Blackman's Formula
- Amplifier Feedback
- Feedback influence on
  - Nonlinearity and noise
  - Bandwidth
- Feedback stability
- Stability simulation
- Other aspects of feedback

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