LECTURE 16 – CURRENT MIRRORS AND SIMPLE REFERENCES

LECTURE ORGANIZATION

Outline
• MOSFET current mirrors
• Improved current mirrors
• Voltage references with power supply independence
• Current references with power supply independence
• Temperature behavior of voltage and current references

CMOS Analog Circuit Design, 3rd Edition Reference
Pages 138-156
MOSFET CURRENT MIRRORS

What is a Current Mirror?

A current mirror replicates the input current of a current sink or current source as an output current. The output current may be identical to the input current or can be a scaled version of it.

\[
i_{\text{OUT}} = K i_{\text{IN}}
\]

The above current mirrors are referenced with respect to ground. Current mirrors can also be referenced with respect to \( V_{DD} \) and can source input and output currents.
Characterization of Current Mirrors

A current mirror is basically nothing more than a current amplifier. The ideal characteristics of a current amplifier are:

- Output current linearly related to the input current, \( i_{out} = A_i i_{in} \)
- Input resistance is zero
- Output resistance is infinity

Also, the characteristic \( V_{MIN} \) applies not only to the output but also the input.

- \( V_{MIN}(in) \) is the range of \( v_{in} \) over which the input resistance is not small
- \( V_{MIN}(out) \) is the range of \( v_{out} \) over which the output resistance is not large

Graphically:

Therefore, \( R_{out} \), \( R_{in} \), \( V_{MIN}(out) \), \( V_{MIN}(in) \), and \( A_i \) will characterize the current mirror.
Simple MOS Current Mirror

Circuit:

Assume that $v_{DS2} > v_{GS} - V_{T2}$, then

$$\frac{i_O}{i_I} = \frac{(L_1 W_2)(V_{GS} - V_{T2})}{(W_1 L_2)(V_{GS} - V_{T1})} \left[ \frac{1 + \lambda v_{DS2}}{1 + \lambda v_{DS1}} \right] \left( \frac{K_2'}{K_1'} \right)$$

If the transistors are matched, then $K_1' = K_2'$ and $V_{T1} = V_{T2}$ to give,

$$\frac{i_O}{i_I} = \frac{(L_1 W_2)}{(W_1 L_2)} \frac{1 + \lambda v_{DS2}}{1 + \lambda v_{DS1}}$$

If $v_{DS1} = v_{DS2}$, then

$$\frac{i_O}{i_I} = \frac{(L_1 W_2)}{(W_1 L_2)}$$

Therefore the sources of error are:

1.) $v_{DS1} \neq v_{DS2}$
2.) M1 and M2 are not matched.
Influence of the Channel Modulation Parameter, $\lambda$

If the transistors are matched and the W/L ratios are equal, then

$$\frac{i_O}{i_I} = \frac{1 + \lambda v_{DS2}}{1 + \lambda v_{DS1}}$$

if the channel modulation parameter is the same for both transistors ($L_1 = L_2$).

Ratio error (%) versus drain voltage difference:

Note that one could use this effect to measure $\lambda$.

Measure $V_{DS1}$, $V_{DS2}$, $i_I$ and $i_O$ and solve the above equation for the channel modulation parameter, $\lambda$.

$$\lambda = \frac{i_O}{i_I} - 1 - \frac{v_{DS2} - \frac{i_O}{i_I} \lambda v_{DS1}}{v_{DS2} - \frac{i_O}{i_I} \lambda v_{DS1}}$$

![Graph showing ratio error vs. drain voltage difference](image-url)
Illustration of the Offset Voltage Error Influence

Assume that $V_{T1} = 0.7\,\text{V}$ and $K'W/L = 110\,\mu\text{A/V}^2$.

Key: Make the part of $V_{GS}$ causing the current to flow, $V_{ON}$, more significant than $V_T$. 
Example 16-1 – Aspect Ratio Errors in Current Mirrors

A layout is shown for a one-to-four current amplifier. Assume that the lengths are identical ($L_1 = L_2$) and find the ratio error if $W_1 = 5 \pm 0.1 \ \mu m$. The actual widths of the two transistors are

$$W_1 = 5 \pm 0.1 \ \mu m \ \text{and} \ W_2 = 20 \pm 0.1 \ \mu m$$

**Solution**

We note that the tolerance is not multiplied by the nominal gain factor of 4.

The ratio of $W_2$ to $W_1$ and consequently the gain of the current amplifier is

$$\frac{i_O}{i_I} = \frac{W_2}{W_1} = \frac{20 \pm 0.1}{5 \pm 0.1} = 4\left(1 \pm \frac{(0.1/20)}{1 \pm (0.1/5)}\right) \approx 4\left(1 \pm \frac{0.1}{20}\right)\left(1 - \frac{0.1}{5}\right) \approx 4\left(1 \pm \frac{0.1}{20} - \frac{0.4}{20}\right) = 4 - (\pm 0.03)$$

where we have assumed that the variations would both have the same sign (correlated). It is seen that this ratio error is 0.75% of the desired current ratio or gain.
Example 16-2 – Reduction of the Aspect Ratio Errors in Current Mirrors

Use the layout technique illustrated below and calculate the ratio error of a current amplifier having the specifications of the previous example.

Solutions

The actual widths of M1 and M2 are

\[ W_1 = 5 \pm 0.1 \, \mu m \quad \text{and} \quad W_2 = 4(5 \pm 0.1) \, \mu m \]

The ratio of \( W_2 \) to \( W_1 \) and consequently the current gain is given below and is for all practical purposes independent of layout error.

\[
\frac{i_O}{i_I} = \frac{4(5 \pm 0.1)}{5 \pm 0.1} = 4
\]
Summary of the Simple MOS Current Mirror/Amplifier

- Minimum input voltage is \( V_{MIN}(in) = V_T + V_{ON} \)
  
  Okay, but could be reduced to \( V_{ON} \).

  Principle:

  Will deal with later in low voltage op amps.

- Minimum output voltage is \( V_{MIN}(out) = V_{ON} \)

- Output resistance is \( R_{out} = \frac{1}{\lambda I_D} \)

- Input resistance is \( R_{in} \approx \frac{1}{g_m} \)

- Current gain accuracy is poor because \( v_{DS1} \neq v_{DS2} \)
IMPROVED CURRENT MIRRORS

Large Output Swing Cascode Current Mirror

- \( R_{out} \approx g_{m2}r_{ds2}r_{ds1} \)
- \( R_{in} = \frac{v_{in}}{i_{in}} = \frac{r_{ds5} + r_{ds3} + r_{ds3}g_{m5}r_{ds5}}{g_{m3}r_{ds3}(1+g_{m5}r_{ds5})} \approx \frac{1}{g_{m3}} \)

An easier way to find \( R_{in} \):

1.) Apply a small voltage change, \( v_{in} \), at the input.
2.) Note that this voltage is equal to \( v_{gs3} \).
3.) This small voltage change causes a current change in the drain of M3 of \( g_{m3}v_{gs3} \) or \( g_{m3}v_{in} \).
4.) The current \( i_{in} \) is equal to \( g_{m3}v_{in} \).
5.) Therefore, dividing \( v_{in} \) by \( i_{in} \) gives \( R_{in} = 1/g_{m3} \).

- \( V_{MIN}(out) = 2V_{ON} \)
- \( V_{MIN}(in) = V_T + V_{ON} \)
- Current gain is excellent because \( V_{DS1} = V_{DS3} \).
Self-Biased Cascode Current Mirror

- \( R_{in} = ? \)

\[
v_{in} = i_{in}R + r_{ds3}(i_{in}-g_{m3}v_{gs3}) + r_{ds1}(i_{in}-g_{m1}v_{gs1})
\]

But,

\[
v_{gs1} = v_{in} - i_{in}R
\]

and

\[
v_{gs3} = v_{in} - r_{ds1}(i_{in} - g_{m1}v_{gs1}) = v_{in} - r_{ds1}i_{in} + g_{m1}r_{ds1}(v_{in} - i_{in}R)
\]

\[
\therefore v_{in} = i_{in}R + r_{ds3}i_{in} - g_{m3}r_{ds3}[v_{in} - r_{ds1}i_{in} + g_{m1}r_{ds1}(v_{in} - i_{in}R)] + r_{ds1}[i_{in} - g_{m1}(v_{in} + i_{in}R)]
\]

\[
v_{in}[1 + g_{m3}r_{ds3} + g_{m1}r_{ds1} + g_{m3}r_{ds3}r_{ds1} + g_{m1}r_{ds1}g_{m3}r_{ds3} + g_{m1}r_{ds1}g_{m3}r_{ds3}R]
\]

\[
R_{in} = \frac{R + r_{ds1} + r_{ds3} + g_{m3}r_{ds3}r_{ds1} + g_{m1}r_{ds1}g_{m3}r_{ds3}R}{1 + g_{m3}r_{ds3} + g_{m1}r_{ds1} + g_{m3}r_{ds3}r_{ds1} + g_{m1}r_{ds1}g_{m3}r_{ds3} + g_{m1}r_{ds1}} \approx \frac{1}{g_{m1}} + R
\]

- \( R_{out} \approx g_{m4}r_{ds4}r_{ds2} \)

- \( V_{MIN}(in) = V_T + 2V_{ON} \)
- \( V_{MIN}(out) = 2V_{ON} \)
- Current gain matching is excellent
MOS Regulated Cascode Current Mirror

- $R_{out} \approx g_m^2 r_{ds}^3$
- $R_{in} \approx \frac{1}{g_m^4}$
- $V_{MIN}(out) = V_T + 2V_{ON}$ (Can be reduced to $2V_{ON}$)
- $V_{MIN}(in) = V_T + V_{ON}$ (Can be reduced to $V_{ON}$)
- Current gain matching - good as long as $v_{DS4} = v_{DS2}$
## Summary of MOS Current Mirrors

<table>
<thead>
<tr>
<th>Current Mirror</th>
<th>Accuracy</th>
<th>Output Resistance</th>
<th>Input Resistance</th>
<th>Minimum Output Voltage</th>
<th>Minimum Input Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>Poor</td>
<td>$r_{ds}$</td>
<td>$\frac{1}{g_m}$</td>
<td>$V_{ON}$</td>
<td>$V_T+V_{ON}$</td>
</tr>
<tr>
<td>Wide Output Swing Cascode</td>
<td>Excellent</td>
<td>$g_m r_{ds}^2$</td>
<td>$\frac{1}{g_m}$</td>
<td>$2V_{ON}$</td>
<td>$V_T+V_{ON}$</td>
</tr>
<tr>
<td>Self-biased Cascode</td>
<td>Excellent</td>
<td>$g_m r_{ds}^2$</td>
<td>$R + \frac{1}{g_m}$</td>
<td>$2V_{ON}$</td>
<td>$V_T+2V_{ON}$</td>
</tr>
<tr>
<td>Regulated Cascode</td>
<td>Good-Excellent</td>
<td>$g_m^2 r_{ds}^3$</td>
<td>$\frac{1}{g_m}$</td>
<td>$V_T+2V_{ON}$ (Can be $2V_{ON}$)</td>
<td>$V_T+V_{ON}$ (Can be $\approx V_{ON}$)</td>
</tr>
</tbody>
</table>
VOLTAGE REFERENCES WITH POWER SUPPLY INDEPENDENCE

Power Supply Independence

How do you characterize power supply independence?

Use the concept of:

$$S_{\frac{V_{REF}}{V_{DD}}} = \frac{V_{REF}/V_{REF}}{V_{DD}/V_{DD}} = \frac{V_{DD}}{V_{REF}} \left( \frac{V_{REF}}{V_{DD}} \right)$$

Application of sensitivity to determining power supply dependence:

$$\frac{V_{REF}}{V_{REF}} = \left( S_{\frac{V_{REF}}{V_{DD}}} \right) \frac{V_{DD}}{V_{DD}}$$

Thus, the fractional change in the reference voltage is equal to the sensitivity times the fractional change in the power supply voltage.

For example, if the sensitivity is 1, then a 10% change in $V_{DD}$ will cause a 10% change in $V_{REF}$.

Ideally, we want $S_{\frac{V_{REF}}{V_{DD}}}$ to be zero for power supply independence.
MOSFET-Resistance Voltage References

Simple MOS-R Voltage Reference

\[ V_{\text{REF}} = V_{GS} = V_T + \sqrt{\frac{2(V_{DD} - V_{\text{REF}})}{\beta R}} \]

or

\[ V_{\text{REF}} = V_T - \frac{1}{\beta R} + \sqrt{\frac{2(V_{DD} - V_T)}{\beta R}} + \frac{1}{(\beta R)^2} \]

\[ S = \frac{V_{\text{REF}}}{V_{DD}} \left( \frac{1}{\sqrt{1 + 2\beta(V_{DD} - V_T)R}} \right) \left( \frac{V_{DD}}{V_{\text{REF}}} \right) \]

Assume \( V_{DD}=5V, W/L =100 \) and \( R=100k\Omega \), thus \( V_{\text{REF}} \approx 0.7875V \) and \( S = 0.0653 \)

Higher Voltage Simple MOS-R Voltage Reference

This circuit allows \( V_{\text{REF}} \) to be larger. If the current in \( R_1 \) (and \( R_2 \)) is small compared to the current flowing through the transistor, then

\[ V_{\text{REF}} \approx \left( \frac{R_1 + R_2}{R_2} \right) V_{GS} \]
Bipolar-Resistance Voltage References

\[ V_{REF} = V_{EB} = \frac{kT}{q} \ln \left( \frac{I}{I_s} \right) \]

and

\[ I = \frac{V_{CC} - V_{EB}}{R} \approx \frac{V_{CC}}{R} \]

give

\[ V_{REF} \approx \frac{kT}{q} \ln \left( \frac{V_{CC}}{RI_s} \right) \]

\[ S_{V_{REF}}^{V_{CC}} = \frac{1}{\ln[V_{CC}/(RI_s)]} = \frac{1}{\ln(I/I_s)} \]

If \( V_{CC} = 5V, R = 4.3k\Omega \) and \( I_s = 1fA \), then \( V_{REF} = 0.719V \).

Also, \( S_{V_{REF}}^{V_{CC}} = 0.0362 \)

If the current in \( R_1 \) (and \( R_2 \)) is small compared to the current flowing through the transistor, then

\[ V_{REF} \approx \left( \frac{R_1 + R_2}{R_1} \right) V_{EB} \]

Can use diodes in place of the BJTs.
CURRENT REFERENCES WITH POWER SUPPLY INDEPENDENCE

Power Supply Independence

Again, we want
\[
S_{V_{DD}}^{I_{REF}} = \frac{I_{REF}/I_{REF}}{V_{DD}/V_{DD}} = \frac{V_{DD}}{I_{REF}} \left( \frac{I_{REF}}{V_{DD}} \right)
\]
to approach zero.

Therefore, as \( S_{V_{DD}}^{I_{REF}} \) approaches zero, the change in \( I_{REF} \) as a function of a change in \( V_{DD} \) approaches zero.
Gate-Source Referenced Current Reference

The circuit below uses both positive and negative feedback to accomplish a current reference that is reasonably independent of power supply.

Circuit:

![Circuit Diagram]

Principle:

If $M3 = M4$, then $I_1 \approx I_2$. However, the M1-R loop gives $V_{GS1} = V_T + \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}$

Solving these two equations gives $I_2 = \frac{V_{GS1}}{R} = \frac{V_T}{R} + \left(\frac{1}{R}\right) \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}$

The output current, $I_{out} = I_1 = I_2$ can be solved as $I_{out} = \frac{V_T}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_T}{\beta_1 R^2} + \frac{1}{(\beta_1 R)^2}}$
Simulation Results for the Gate-Source Referenced Current Reference

The current $I_{D2}$ appears to be okay, why is $I_{D1}$ increasing?
Apparently, the channel modulation on the current mirror M3-M4 is large.
At $V_{DD} = 5\text{V}$, $V_{SD3} = 2.83\text{V}$ and $V_{SD4} = 1.09\text{V}$ which gives $I_{D3} = 1.067I_{D4} \approx 107\mu\text{A}$
Need to cascode the upper current mirror.

SPICE Input File:

```
Simple, Bootstrap Current Reference
VDD 1 0 DC 5.0
VSS 9 0 DC 0.0
M1 5 7 9 9 N W=20U L=1U
M2 3 5 7 9 N W=20U L=1U
M3 5 3 1 1 P W=25U L=1U
M4 3 3 1 1 P W=25U L=1U
M5 9 3 1 1 P W=25U L=1U
R 7 9 10KILOHM
M8 6 6 9 9 N W=1U L=1U
M7 6 6 5 9 N W=20U L=1U

RB 1 6 100KILOHM
.OP
.DC VDD 0 5 0.1
.MODEL N NMOS VTO=0.7 KP=110U GAMMA=0.4 +PHI=0.7 LAMBDA=0.04
.MODEL P PMOS VTO=-0.7 KP=50U GAMMA=0.57 +PHI=0.8 LAMBDA=0.05
.PRINT DC ID(M1) ID(M2) ID(M5)
.PROBE
.END
```
Cascoded Gate-Source Referenced Current Reference

SPICE Input File:

```
Cascode, Bootstrap Current Reference
VDD 1 0 DC 5.0
VSS 9 0 DC 0.0
M1 5 7 9 9 N W=20U L=1U
M2 4 5 7 9 N W=20U L=1U
M3 2 3 1 1 P W=25U L=1U
M4 8 3 1 1 P W=25U L=1U
M3C 5 4 2 1 P W=25U L=1U
MC4 3 4 8 1 P W=25U L=1U
RON 3 4 4KILOHM
M5 9 3 1 1 P W=25U L=1U
R 7 9 10KILOHM

M8 6 6 9 9 N W=1U L=1U
M7 6 6 5 9 N W=20U L=1U
RB 1 6 100KILOHM
.OP
.DC VDD 0 5 0.1
.MODEL N NMOS VTO=0.7 KP=110U GAMMA=0.4 PHI=0.7 LAMBDA=0.04
.MODEL P PMOS VTO=-0.7 KP=50U GAMMA=0.57 PHI=0.8 LAMBDA=0.05
.PRINT DC ID(M1) ID(M2) ID(M5)
.PROBE
.END
```

Fig. 370-08
Base-Emitter Referenced Circuit

\[ I_{out} = I_2 = \frac{V_{EB1}}{R} \]

BJT can be a MOSFET in weak inversion.
Low Voltage Gate-Source Referenced MOS Current Reference

The previous gate-source referenced circuits required at least 2 volts across the power supply before operating.

A low-voltage gate-source referenced circuit:

Without the batteries, \( V_T \), the minimum power supply is \( V_T + 2V_{ON} + V_R \).

With the batteries, \( V_T \), the minimum power supply is \( 2V_{ON} + V_R \approx 0.5V \)
Summary of Power-Supply Independent References

- Reasonably good, simple voltage and current references are possible
- Best power supply sensitivity is approximately 0.01
  (10% change in power supply causes a 0.1% change in reference)

<table>
<thead>
<tr>
<th>Type of Reference</th>
<th>$V_{REF}$ or $S_{V_{PP}}$</th>
<th>$I_{REF}$ or $S_{V_{PP}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSFET-R</td>
<td>&lt;1</td>
<td></td>
</tr>
<tr>
<td>BJT-R</td>
<td>&lt;&lt;1</td>
<td></td>
</tr>
<tr>
<td>Gate-source Referenced</td>
<td>&lt;&lt;1</td>
<td></td>
</tr>
<tr>
<td>Base-emitter Referenced</td>
<td>&lt;&lt;1</td>
<td></td>
</tr>
</tbody>
</table>
TEMPERATURE BEHAVIOR OF VOLTAGE AND CURRENT REFERENCES

Characterization of Temperature Dependence

The objective is to minimize the fractional temperature coefficient defined as,

\[ TC_F = \frac{1}{V_{\text{REF}}} \left( \frac{V_{\text{REF}}}{T} \right) = \frac{1}{T} S \frac{V_{\text{REF}}}{T} \text{ parts per million per } ^\circ \text{C or ppm/}^\circ \text{C} \]

Temperature dependence of PN junctions:

\[ i \approx I_s \exp \left( \frac{v}{V_t} \right) \]

\[ I_s = K T^3 \exp \left( -\frac{V_{GO}}{V_t} \right) \]

\[ \frac{dv_{BE}}{dT} \approx \frac{V_{BE} - V_{GO}}{T} = -2 \text{mV/}^\circ \text{C at room temperature} \]

Temperature dependence of MOSFET in strong inversion:

\[ \frac{dv_{GS}}{dT} = \frac{dV_T}{dT} + \sqrt{\frac{2L}{W C_{\text{ox}}}} \frac{d}{dT} \left( \sqrt{\frac{i_D}{\mu_o}} \right) \]

\[ \mu_o = K T^{-1.5} \]

\[ V_T(T) = V_T(T_o) - \alpha(T-T_o) \]

Resistors: \((1/R)(dR/dT)\) ppm/°C
**Bipolar-Resistance Voltage References**

From previous work we know that,

\[ V_{\text{REF}} = \frac{kT}{q} \ln \left( \frac{V_{\text{DD}} - V_{\text{REF}}}{RI_s} \right) \]

However, not only is \( V_{\text{REF}} \) a function of \( T \), but \( R \) and \( I_s \) are also functions of \( T \).

\[
\frac{dV_{\text{REF}}}{dT} = \frac{k}{q} \ln \left( \frac{V_{\text{DD}} - V_{\text{REF}}}{RI_s} \right) - \frac{kT}{RI_s} - \left( \frac{V_{\text{DD}} - V_{\text{REF}}}{RI_s} \right) \left( \frac{dR}{RdT} + \frac{dI_s}{I_sdT} \right)
\]

\[
= \frac{V_{\text{REF}}}{T} - \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} - V_t \left( \frac{dR}{RdT} + \frac{dI_s}{I_sdT} \right) = \frac{V_{\text{REF}} - V_{\text{GO}}}{T} - \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} - \frac{3V_t}{T} \frac{dR}{RdT}
\]

\[
\frac{dV_{\text{REF}}}{dT} = \frac{V_{\text{REF}} - V_{\text{GO}}}{T} - \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} - \frac{3V_t}{T} \frac{dR}{RdT} - \frac{3V_t}{T}
\]

\[
TC_F = \frac{1}{V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} = \frac{V_{\text{REF}} - V_{\text{GO}}}{V_{\text{REF}} \cdot T} - \frac{V_t}{V_{\text{REF}}} \frac{dR}{RdT} - \frac{3V_t}{V_{\text{REF}} \cdot T}
\]

If \( V_{\text{REF}} = 0.6 \text{V} \), \( V_t = 0.026 \text{V} \), and the \( R \) is polysilicon, then at 27°K the \( TC_F \) is

\[
TC_F = \frac{0.6 - 1.205}{0.6 \cdot 300} - \frac{0.026 \cdot 0.0015}{0.6} - \frac{3 \cdot 0.026}{0.6 \cdot 300} = 33110^{-6} - 65 \times 10^{-6} - 433 \times 10^{-6} = -3859 \text{ppm/°C}
\]
MOSFET Resistor Voltage Reference

From previous results we know that

\[ V_{REF} = V_{GS} = V_T + \sqrt{\frac{2(V_{DD} - V_{REF})}{\beta R}} \]

or

\[ V_{REF} = V_T - \frac{1}{\beta R} + \sqrt{\frac{2(V_{DD} - V_T)}{\beta R} + \frac{1}{(\beta R)^2}} \]

Note that \( V_{REF}, V_T, \beta, \) and \( R \) are all functions of temperature.

It can be shown that the \( TCF \) of this reference is

\[
\frac{dV_{REF}}{dT} = -\alpha + \sqrt{\frac{V_{DD} - V_{REF}}{2\beta R}} \left( \frac{1.5}{T} - \frac{1}{R} \frac{dR}{dT} \right) \\
1 + \frac{1}{\sqrt{2\beta R (V_{DD} - V_{REF})}} \\
\]

\[ TCF = \frac{V_{REF}(1 + \frac{1}{\sqrt{2\beta R (V_{DD} - V_{REF})}})}{\frac{1.5}{T} - \frac{1}{R} \frac{dR}{dT}} \]
Example 16-3 - Calculation of MOSFET-Resistor Voltage Reference $TC_F$

Calculate the temperature coefficient of the MOSFET-Resistor voltage reference where $W/L=2$, $V_{DD}=5V$, $R=100k\Omega$ using the parameters of Table 3.1-2. The resistor, $R$, is polysilicon and has a temperature coefficient of 1500 ppm/°C.

**Solution**

First, calculate $V_{REF}$. Note that $\beta R = 220 \times 10^{-6} \times 10^5 = 22$ and $\frac{dR}{RdT} = 1500 \text{ppm}/^\circ\text{C}$

\[
V_{REF} = 0.7 - \frac{1}{22} + \sqrt{\frac{2(5 - 0.7)}{22}} + \left(\frac{1}{22}\right)^2 = 1.281\text{V}
\]

Now, $\frac{dV_{REF}}{dT} = -\frac{2.3 \times 10^{-3}}{1 + \frac{1}{\sqrt{2(22)}}} + \sqrt{\frac{5 - 1.281}{2(22)}} \left(\frac{1.5}{300} - 1500x10^{-6}\right) = -1.189 \times 10^{-3}\text{V}/^\circ\text{C}$

The fractional temperature coefficient is given by

\[
TC_F = -1.189 \times 10^{-3} \left(\frac{1}{1.281}\right) = -928 \text{ ppm}/^\circ\text{C}
\]
Gate-Source and Base-Emitter Referenced Current Source/Sinks

Gate-source referenced source:

The output current was given as, \( I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R} + \frac{1}{(\beta_1 R)^2}} \)

Although we could grind out the derivative of \( I_{out} \) with respect to \( T \), the temperature performance of this circuit is not that good to spend the time to do so. Therefore, let us assume that \( V_{GS1} \approx V_{T1} \) which gives

\[
I_{out} \approx \frac{V_{T1}}{R} \quad \Rightarrow \quad \frac{dI_{out}}{dT} = \frac{1}{R} \frac{dV_{T1}}{dT} - \frac{1}{R^2} \frac{dR}{dT}
\]

In the resistor is polysilicon, then

\[
TC_F = \frac{1}{I_{out}} \frac{dI_{out}}{dT} = \frac{1}{V_{T1}} \frac{dV_{T1}}{dT} - \frac{1}{R} \frac{dR}{dT} = \frac{-\alpha}{V_{T1}} \frac{1}{R} \frac{dR}{dT} = \frac{-2.3 \times 10^{-3}}{0.7} - 1.5 \times 10^{-3} = -4786 \text{ppm/°C}
\]

Base-emitter referenced source:

The output current was given as, \( I_{out} = I_2 = \frac{V_{BE1}}{R} \)

The \( TC_F = \frac{1}{V_{BE1}} \frac{dV_{BE1}}{dT} - \frac{1}{R} \frac{dR}{dT} \)

If \( V_{BE1} = 0.6 \text{V} \) and \( R \) is poly, then the \( TC_F = \frac{1}{0.6} (-2 \times 10^{-3}) - 1.5 \times 10^{-3} = -4833 \text{ppm/°C} \).
Low $V_{DD}$ Current Reference

Consider the following circuit with all transistors having a $W/L = 10$. This is a bootstrapped reference which creates a $V_{bias}$ independent of $V_{DD}$. The two key equations are:

$$I_3 = I_4 \Rightarrow I_1 = I_2$$

and

$$V_{GS1} = V_{GS2} + I_2R$$

Solving for $I_2$ gives:

$$I_2 = \frac{V_{GS1} - V_{GS2}}{R} = \frac{1}{R} \left( \sqrt{\frac{2I_1}{\beta_1}} - \sqrt{\frac{2I_2}{\beta_2}} \right) = \frac{\sqrt{2I_1}}{R} \frac{1 - \frac{1}{2}}{\beta_1}$$

$$\therefore \sqrt{I_2} = \frac{1}{R\sqrt{2\beta_1}} \Rightarrow I_2 = I_1 = \frac{1}{2\beta_1R^2} = \frac{1}{2 \cdot 110 \times 10^{-6} \cdot 10 \cdot 25 \times 10^6} = 18.18 \mu A$$

Now, $V_{bias}$ can be written as

$$V_{bias} = V_{GS1} = \sqrt{\frac{2I_2}{\beta_1}} + V_{TN} = \frac{1}{\beta_1R} + V_{TN} = \frac{1}{110 \times 10^{-6} \cdot 10 \cdot 5 \times 10^3} + 0.7 = 0.1818 + 0.7 = 0.8818V$$

Any transistor with $V_{GS} = V_{bias}$ will have a current flow that is given by $1/2\beta R^2$.

Therefore,

$$g_m = \sqrt{2I\beta} = \sqrt{\frac{2\beta}{2\beta R^2}} = \frac{1}{R} \Rightarrow g_m = \frac{1}{R}$$
Summary of Reference Performance

<table>
<thead>
<tr>
<th>Type of Reference</th>
<th>$S_{\frac{V_{REF}}{V_{DD}}}$</th>
<th>$TC_F$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSFET-R</td>
<td>$&lt;1$</td>
<td>$&gt;1000$ppm/$\degree$C</td>
<td></td>
</tr>
<tr>
<td>BJT-R</td>
<td>$&lt;&lt;1$</td>
<td>$&gt;1000$ppm/$\degree$C</td>
<td></td>
</tr>
<tr>
<td>Gate-Source Referenced</td>
<td>Good if currents are matched</td>
<td>$&gt;1000$ppm/$\degree$C</td>
<td>Requires start-up circuit</td>
</tr>
<tr>
<td>Base-emitter Referenced</td>
<td>Good if currents are matched</td>
<td>$&gt;1000$ppm/$\degree$C</td>
<td>Requires start-up circuit</td>
</tr>
</tbody>
</table>

- A MOSFET can have zero temperature dependence of $i_D$ for a certain $v_{GS}$
- If one is careful, very good independence of power supply can be achieved
- None of the above references have really good temperature independence

Consider the following example:

A 10 bit ADC has a reference voltage of 1V. The LSB is approximately 0.001V. Therefore, the voltage reference must be stable to within 0.1%. If a 100°C change in temperature is experienced, then the $TC_F$ must be 0.001%/C or multiplying by $10^4$ requires a $TC_F = 10$ppm/$\degree$C.