LECTURE 18 – INVERTING AMPLIFIERS

LECTURE ORGANIZATION

Outline

- Introduction
- Active Load Inverting Amplifier
- Current Source Load Inverting Amplifier
- Push-Pull Inverting Amplifier
- Noise Analysis of Inverting Amplifiers
- Summary

***CMOS Analog Circuit Design*, 3rd Edition Reference**

Pages 186-198

INTRODUCTION

Types of Amplifiers

Type of Amplifier	Gain = $\frac{\text{Output}}{\text{Input}}$	Ideal Input Resistance	Ideal Output Resistance
Voltage	$A_v = \frac{\text{Output Voltage}}{\text{Input Voltage}}$	Infinite	Zero
Current	$A_i = \frac{\text{Output Current}}{\text{Input Current}}$	Zero	Infinite
Transconductance	$G_m = \frac{\text{Output Current}}{\text{Input Voltage}}$	Infinite	Infinite
Transresistance	$R_m = \frac{\text{Output Voltage}}{\text{Input Current}}$	Zero	Zero

Most CMOS amplifiers fit naturally into the transconductance amplifier category as they have large input resistance and fairly large output resistance.

If the load resistance is high, the CMOS transconductance amplifier is essentially a voltage amplifier.

Characterization of an Amplifier

1.) Large signal static characterization:

- Plot of output versus input (transfer curve)
- Large signal gain
- Output and input swing limits

2.) Small signal static characterization:

- AC gain
- AC input resistance
- AC output resistance

3.) Small signal dynamic characterization:

- Bandwidth
- Noise
- Power supply rejection

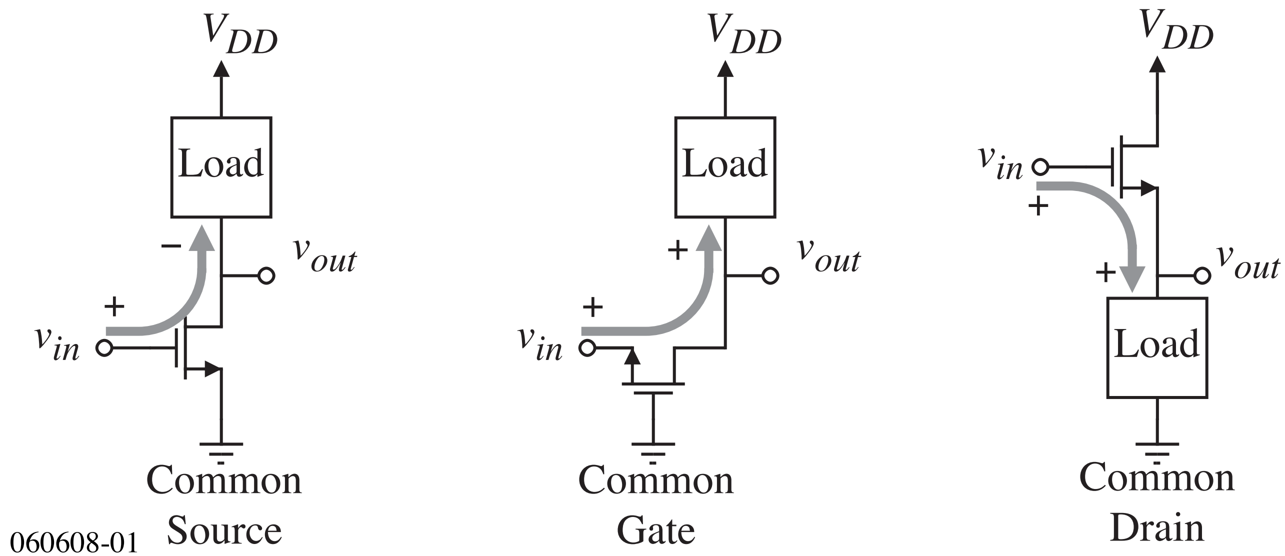
4.) Large signal dynamic characterization:

- Slew rate
- Nonlinearity

Inverting and Noninverting Amplifiers

The types of amplifiers are based on the various configurations of the actual transistors.

If we assume that one terminal of the transistor is grounded, then three possibilities result:



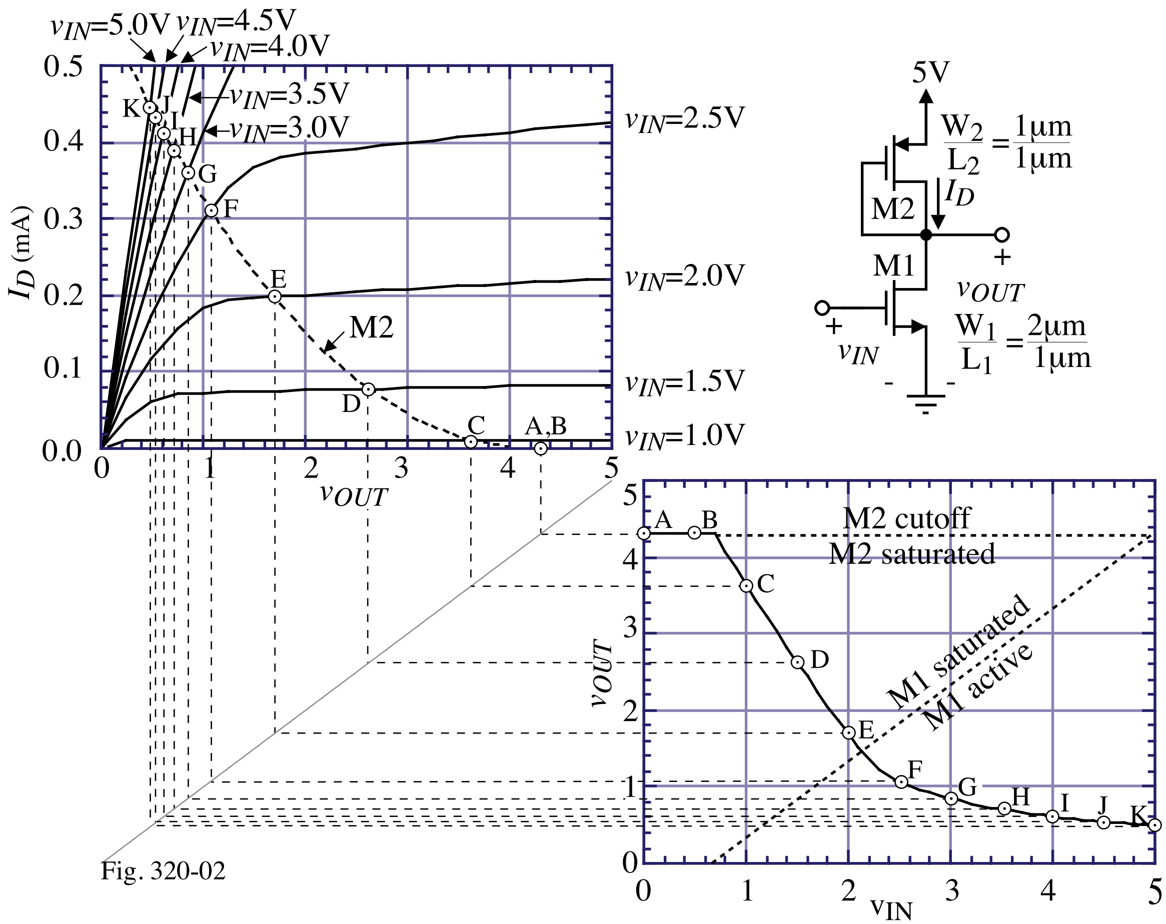
Note that there are two categories of amplifiers:

1.) Noninverting - Those whose input and output are in phase (common gate and common drain)

2.) Inverting - Those whose input and output are out of phase (common source)

ACTIVE LOAD INVERTING AMPLIFIER

Voltage Transfer Characteristic of the Active Load Inverter



The boundary between active and saturation operation for M1 is

$$v_{DS1} \geq v_{GS1} - V_{TN} \quad \rightarrow \quad v_{OUT} \geq v_{IN} - 0.7\text{V}$$

Large-Signal Voltage Swing Limits of the Active Load Inverter

Maximum output voltage, v_{OUT}(max):

$$v_{OUT}(\max) \cong V_{DD} - |V_{TP}|$$

(ignores subthreshold current influence on the MOSFET)

Minimum output voltage, v_{OUT}(min):

Assume that M1 is nonsaturated and that $V_{T1} = |V_{T2}| = V_T$.

$$v_{DS1} \geq v_{GS1} - V_{TN} \quad \rightarrow \quad v_{OUT} \geq v_{IN} - 0.7\text{V}$$

The current through M1 is

$$i_D = \beta_1\left((v_{GS1} - V_T)v_{DS1} - \frac{v_{DS1}^2}{2}\right) = \beta_1\left((V_{DD} - V_T)(v_{OUT}) - \frac{(v_{OUT})^2}{2}\right)$$

and the current through M2 is

$$i_D = \frac{\beta_2}{2}(v_{SG2} - V_T)^2 = \frac{\beta_2}{2}(V_{DD} - v_{OUT} - V_T)^2 = \frac{\beta_2}{2}(v_{OUT} + V_T - V_{DD})^2$$

Equating these currents gives the minimum v_{OUT} as,

$$v_{OUT}(\min) = V_{DD} - V_T - \frac{V_{DD} - V_T}{\sqrt{1 + (\beta_2/\beta_1)}}$$

Small-Signal Midband Performance of the Active Load Inverter

The development of the small-signal model for the active load inverter is shown below:

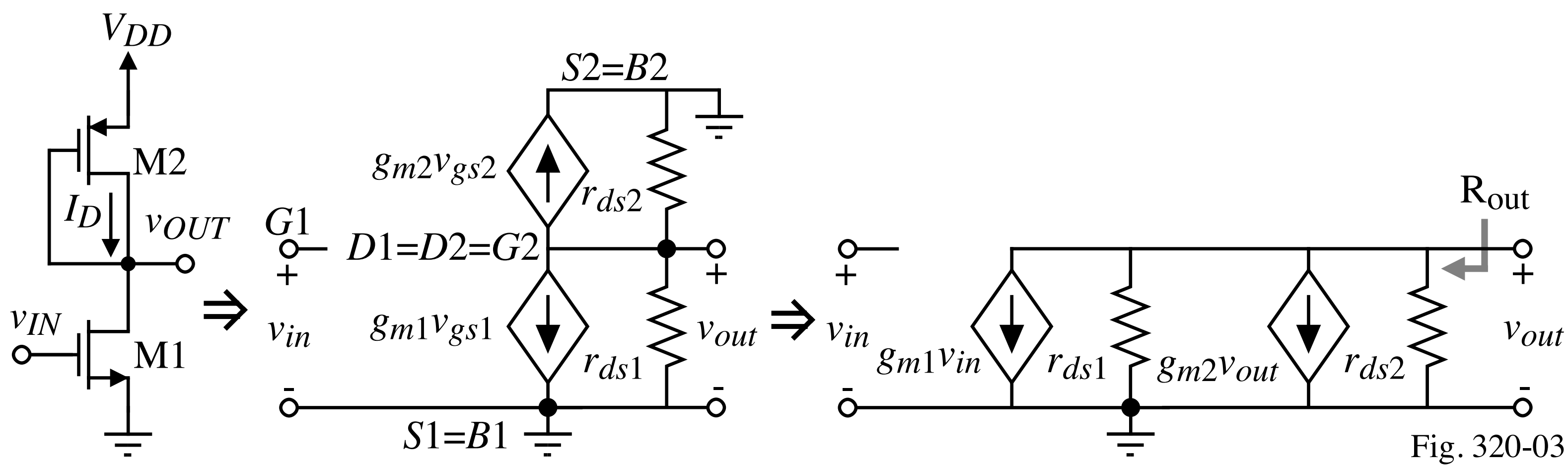


Fig. 320-03

Sum the currents at the output node to get,

$$g_{m1}v_{in} + g_{ds1}v_{out} + g_{m2}v_{out} + g_{ds2}v_{out} = 0$$

Solving for the voltage gain, v_{out}/v_{in}, gives

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{-g_{m1}}{g_{ds1} + g_{ds2} + g_{m2}} \cong -\frac{g_{m1}}{g_{m2}} = -\left(\frac{K'_N W_1 L_2}{K'_P L_1 W_2}\right)^{1/2}$$

The small-signal output resistance can also be found from the above by letting $v_{in} = 0$ to get,

$$R_{\text{out}} = \frac{1}{g_{ds1} + g_{ds2} + g_{m2}} \cong \frac{1}{g_{m2}}$$

Frequency Response of the Active Load Inverter

Incorporation of the parasitic capacitors into the small-signal model:

If we assume the input voltage has a small source resistance, then we can write the following:

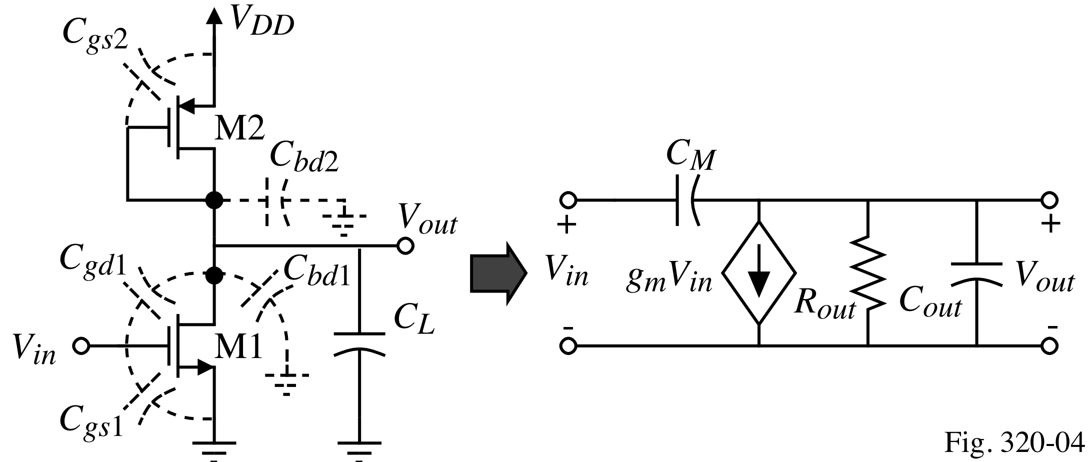


Fig. 320-04

$$sC_M(V_{out}-V_{in}) + g_mV_{in} + G_{out}V_{out} + sC_{out}V_{out} = 0$$

$$\therefore \quad V_{out}(G_{out} + sC_M + sC_{out}) = -(g_m - sC_M)V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{-(g_m - sC_M)}{G_{out} + sC_M + sC_{out}} = -g_mR_{out}\left[\frac{1-\frac{sC_M}{g_m}}{1+sR_{out}(C_M + C_{out})}\right] = \frac{-g_mR_{out}\left(1-\frac{s}{z_1}\right)}{1-\frac{s}{p_1}}$$

where $g_m = g_{m1}$, $p_1 = \dfrac{-1}{R_{out}(C_{out}+C_M)}$, and $z_1 = \dfrac{g_{m1}}{C_M}$

and $R_{out} = [g_{ds1}+g_{ds2}+g_{m2}]^{-1} \cong \dfrac{1}{g_{m2}}$, $C_M = C_{gd1}$, and $C_{out} = C_{bd1}+C_{bd2}+C_{gs2}+C_L$

Complex Frequency (s) Analysis of Circuits – (Optional)

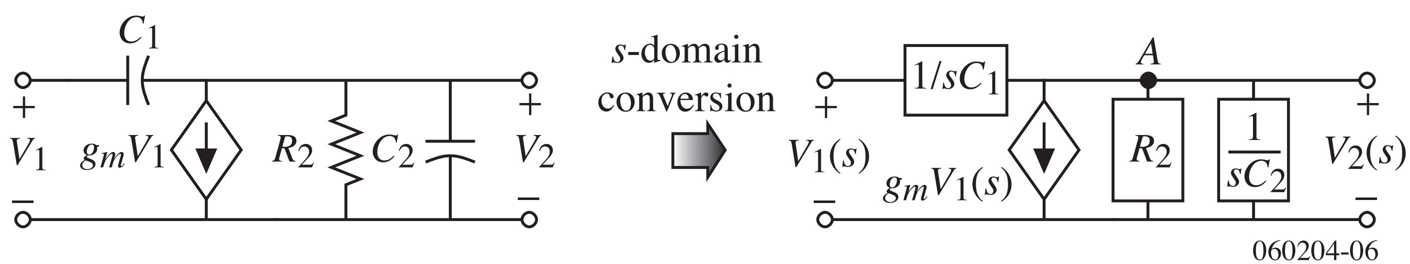
The frequency response of linear circuits can be analyzed using the complex frequency variable s which avoids having to solve the circuit in the time domain and then transform into the frequency domain.

Passive components in the s domain are:

$$Z_R(s) = R \qquad Z_L(s) = sL \qquad \text{and} \qquad Z_C(s) = \frac{1}{sC}$$

s-domain analysis uses the complex impedance of elements as if they were "resistors".

Example:



Sum currents flowing away from node A to get,

$$sC_1(V_2 - V_1) + g_mV_1 + G_2V_2 + sC_2V_2 = 0$$

Solving for the voltage gain transfer function gives,

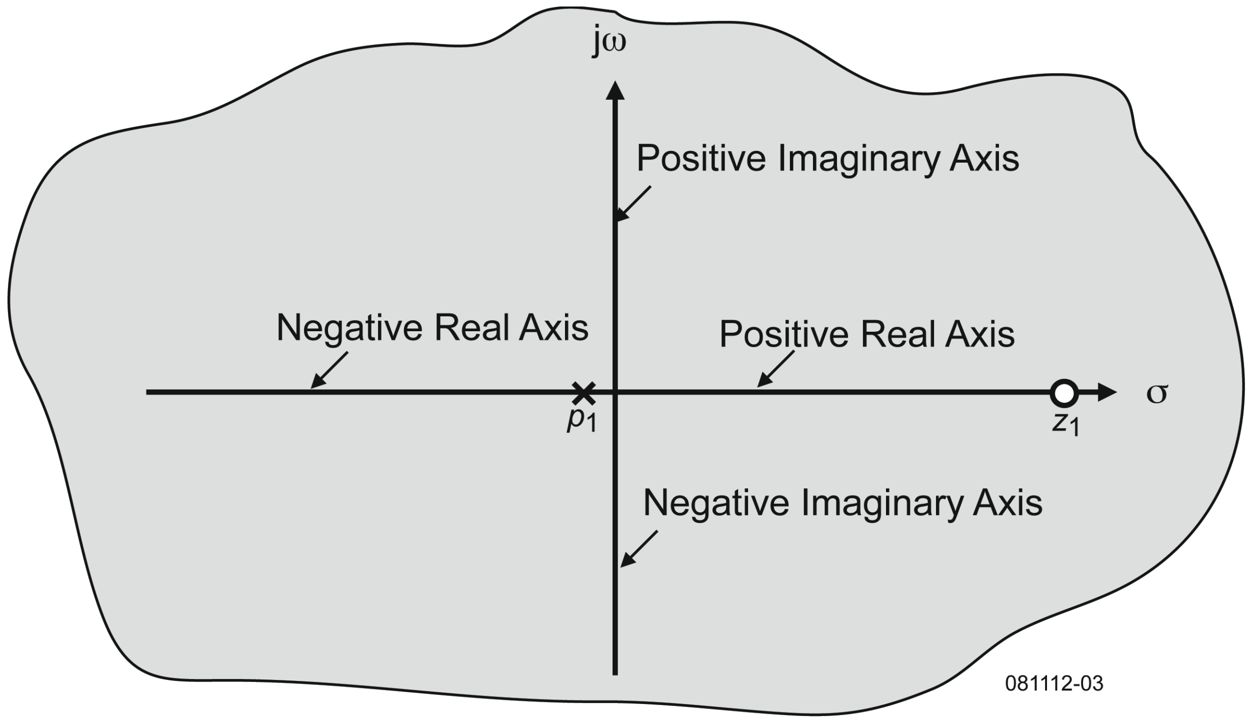
$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-sC_1 + g_m}{s(C_1 + C_2) + G_2} = -g_mR_2\left(\frac{sC_1/g_m - 1}{s(C_1 + C_2)R_2 + 1}\right)$$

Complex Frequency Plane – (Optional)

The complex frequency variable, s, is really a complex number and can be expressed as

$s = \sigma + j\omega$ where $\sigma = \mathrm{Re}[s]$ and $\omega = \mathrm{Im}[s]$.

Complex frequency plane:



It is useful to plot the roots of the transfer function on the complex frequency plane. For the previous $T(s)$, the roots are:

The numerator root (zero) is $s = z_1 = +(g_m/C_1)$

The denominator root (pole) is $s = p_1 = -[1/R_2(C_1 + C_2)]$

What is the Frequency Response of an Amplifier? – (Optional)

Frequency response results when we replace the complex frequency variable s with $j\omega$ in the transfer function of an amplifier. (This amounts to evaluating $T(s)$ on the imaginary axis of the complex frequency plane.)

The frequency response is characterized by the magnitude and phase of $T(j\omega)$.

Example:

$$\text{Assume } T(s) = \frac{a_0 + a_1 s}{b_0 + b_1 s} \quad \xrightarrow{s = j\omega} \quad T(j\omega) = \frac{a_0 + a_1 j\omega}{b_0 + b_1 j\omega} = \frac{a_0 + j\omega a_1}{b_0 + j\omega\, b_1}$$

Since $T(j\omega)$ is a complex number, we can express the magnitude and phase as,

$$|T(j\omega)| = \sqrt{\frac{a_0{}^2 + (\omega a_1)^2}{b_0{}^2 + (\omega\, b_1)^2}} \qquad \text{Arg}[T(j\omega)] = +\tan^{-1}\left(\frac{\omega a_1}{a_0}\right) - \tan^{-1}\left(\frac{\omega b_1}{b_0}\right)$$

For the previous example, the magnitude and phase would be,

$$|T(j\omega)| = g_m R_2 \sqrt{\frac{1 + (\omega C_1/g_m)^2}{1 + [\omega\, R_2(C_1 + C_2)]^2}}$$

$$\text{Arg}[T(j\omega)] = -\tan^{-1}(\omega C_1/g_m) - \tan^{-1}[\omega\, R_2(C_1 + C_2)]$$

Note: Because the zero is on the positive real axis, the phase due to the zero is $-\tan^{-1}(\ )$ rather than $+\tan^{-1}(\ )$. More about that later.

Linear Graphical Illustration of Magnitude and Phase – (Optional)

The important concepts of frequency response are communicated through the graphical portrayal of the magnitude and phase.

Consider our example,

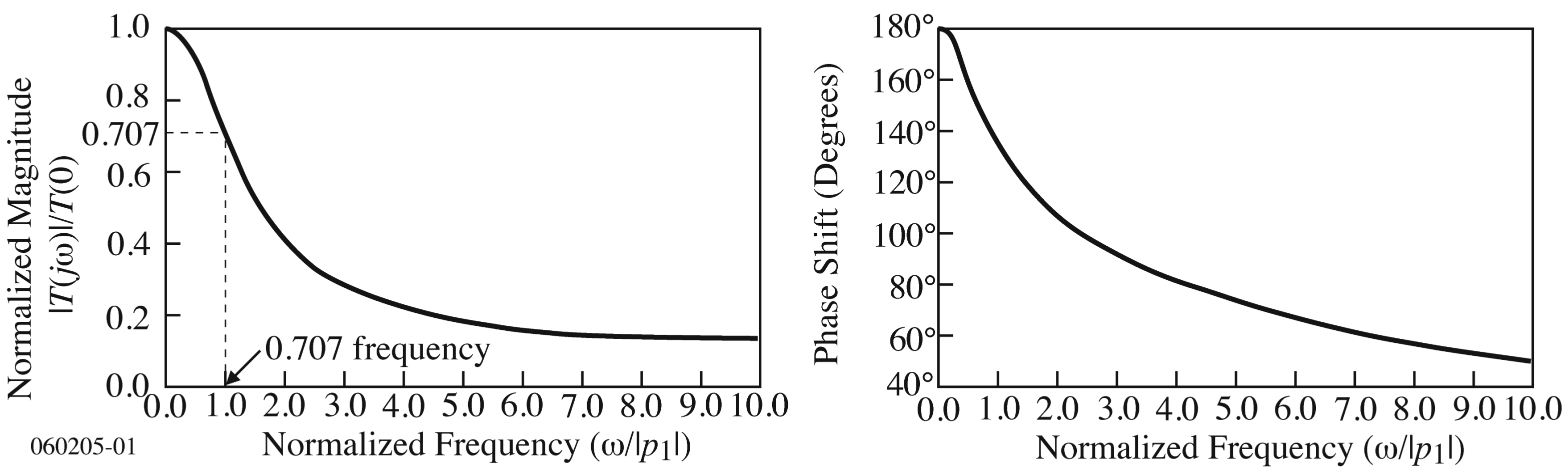
$$T(s) = \frac{V_2(s)}{V_1(s)} = -g_m R_2 \left(\frac{sC_1/g_m - 1}{s(C_1+C_2)R_2+1}\right) = -T(0)\left(\frac{s/z_1 - 1}{s/p_1 - 1}\right)$$

where $T(0) = g_m R_2$, $z_1 = +(g_m/C_1)$ and $p_1 = -[1/R_2(C_1+C_2)]$.

Replacing s with $j\omega$ gives [remember $\tan^{-1}(-x) = -\tan^{-1}(x)$],

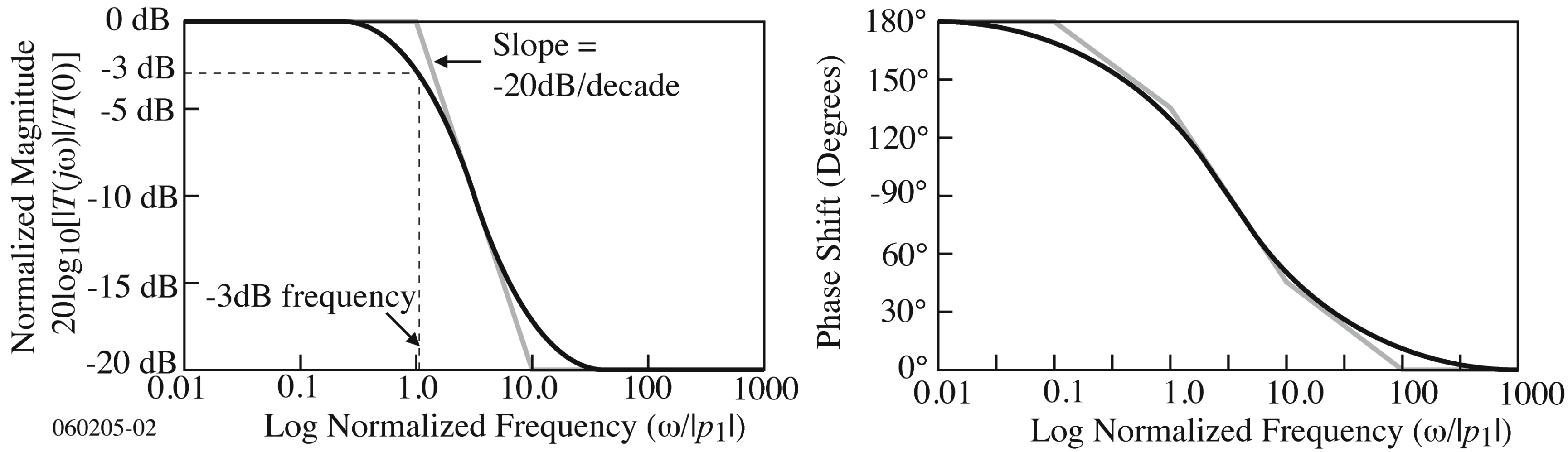
$$|T(j\omega)| = T(0)\sqrt{\frac{1+(\omega/z_1)^2}{1+(\omega/p_1)^2}} \quad \text{and} \quad \text{Arg}[T(j\omega)] = \pm 180° - \tan^{-1}(\omega/z_1) - \tan^{-1}[\omega/p_1]$$

Graphically, we get the following if we assume $|p_1| = 0.1|z_1|$,



Logarithmic Graphical Illustration of Frequency Response – (Optional)

If the frequency range is large, it is more useful to use a logarithmic scale for the frequency. In addition, if one expresses the magnitude as 20 $\log_{10}(|T(j\omega)|$, the plots can be closely approximated with straight lines which enables quick analysis by hand. Such plots are called *Bode plots*.



To construct a Bode asymptotic magnitude plot for a low pass transfer function in the form of products of roots:

1.) Start at a low frequency and plot 20 $\log_{10}(|T(0)|$ until you reach the smallest root.

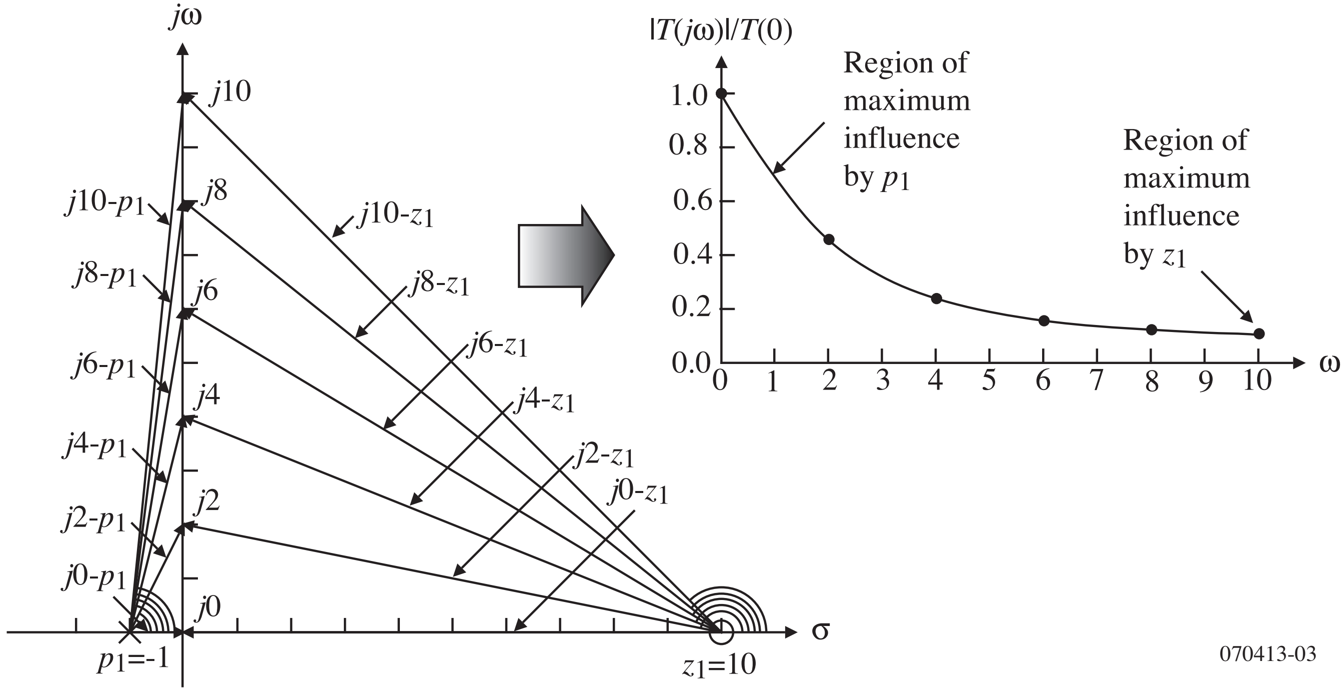
2.) At the frequency equal to magnitude of the smallest root, change to a line with a slope of +20dB/decade if the root is a zero or -20dB/decade if the root is a pole.

3.) Continue increasing in frequency until you have plotted the influence of all roots.

Influence of the Complex Frequency Plane on Frequency Response – (Optional)

The root locations in the complex frequency plane have a direct influence on the frequency response as illustrated below. Consider the transfer function:

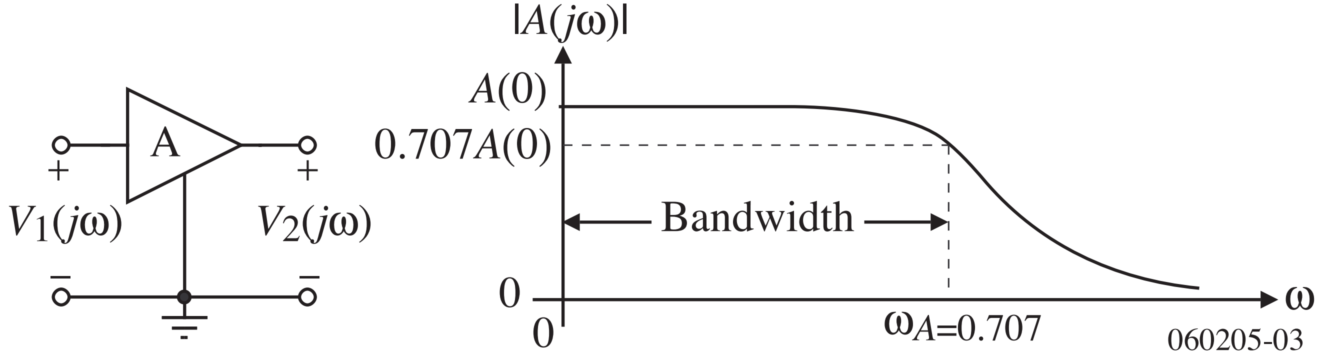
$$T(s) = -T(0)\left(\frac{s/z_1 - 1}{s/p_1 - 1}\right) = -\frac{|p_1|}{z_1}T(0)\left(\frac{s-z_1}{s-p_1}\right) = -0.1T(0)\left(\frac{s-z_1}{s-p_1}\right) \text{ where } z_1 = 10|p_1|$$



Note: The roots maximally influence the magnitude when ω is such that the angle between the vector and the horizontal axis is 45°. This occurs at $j1$ for p_1 and $j10$ for z_1.

Bandwidth of a Low-Pass Amplifier – (Optional)

One of the most important aspects of frequency analysis is to find the frequency at which the amplitude decreases by -3dB or $1/\sqrt{2}$. This can easily be found from the magnitude of the frequency response.



Amplifier with a Dominant Root:

Since the amplifier is low-pass, the poles will be smaller in magnitude than the zeros. If one of the poles is approximately 4-5 times smaller than the next smallest pole, the bandwidth of the amplifier is given as

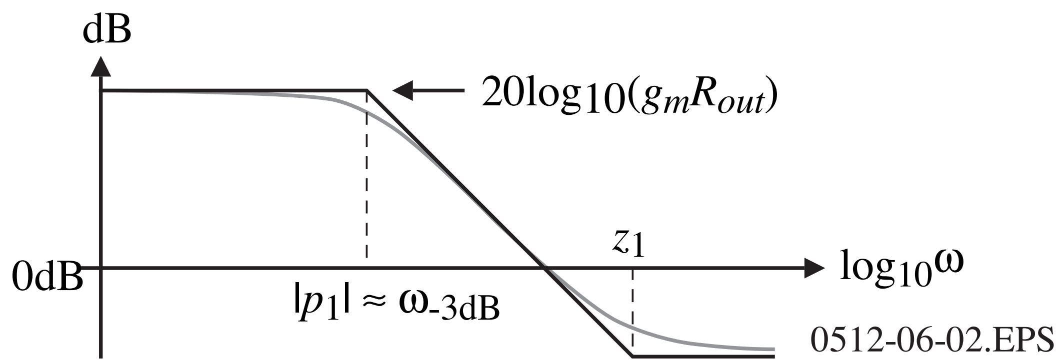
Bandwidth ≈ |Smallest pole|

Amplifier with no Dominant Root:

If there are several poles with roughly the same magnitude, then one should use the graphical method above to find the bandwidth.

Frequency Response of the Active Inverter - Continued

So, back to the frequency response of the active load inverter, we find that if $|p_1| < z_1$, then the -3dB frequency is approximately equal to the magnitude of the pole which is $[R_{out}(C_{out}+C_M)]^{-1}$.



Observation:

In general, the poles in a MOSFET circuit can be found by summing the capacitance connected to a node and multiplying this capacitance times the equivalent resistance from this node to ground and inverting the product.

Example 18-1 - Performance of an Active Load Inverter

Calculate the output-voltage swing limits for V_{DD} = 5 volts, the small-signal gain, the output resistance, and the -3 dB frequency of active load inverter if (W_1/L_1) is 2 μm/1 μm and W_2/L_2 = 1 μm/1 μm, C_{gd1} = 100fF, C_{bd1} = 200fF, C_{bd2} = 100fF, C_{gs2} = 200fF, C_L = 1 pF, and $I_{D1} = I_{D2} = 100\mu$A, using the parameters in Table 3.1-2.

Solution

From the above results we find that:

v_{OUT}(max) = 4.3 volts

v_{OUT}(min) = 0.418 volts

Small-signal voltage gain = -1.92V/V

R_{out} = 9.17 kΩ including g_{ds1} and g_{ds2} and 10 kΩ ignoring g_{ds1} and g_{ds2}

z_1 = 2.10x10^9 rads/sec

p_1 = -64.1x10^6 rads/sec.

Thus, the -3 dB frequency is 10.2 MHz.

CURRENT SOURCE INVERTER

Voltage Transfer Characteristic of the Current Source Inverter

Regions of operation for the transistors:

M1: $v_{DS1} \geq v_{GS1} - V_{Tn}$

or

$$v_{OUT} \geq v_{IN} - 0.7\text{V}$$

M2: $v_{SD2} \geq v_{SG2} - |V_{Tp}|$

or

$$V_{DD} - v_{OUT} \geq V_{DD} - V_{GG2} - |V_{Tp}|$$

or

$$v_{OUT} \leq 3.2\text{V}$$

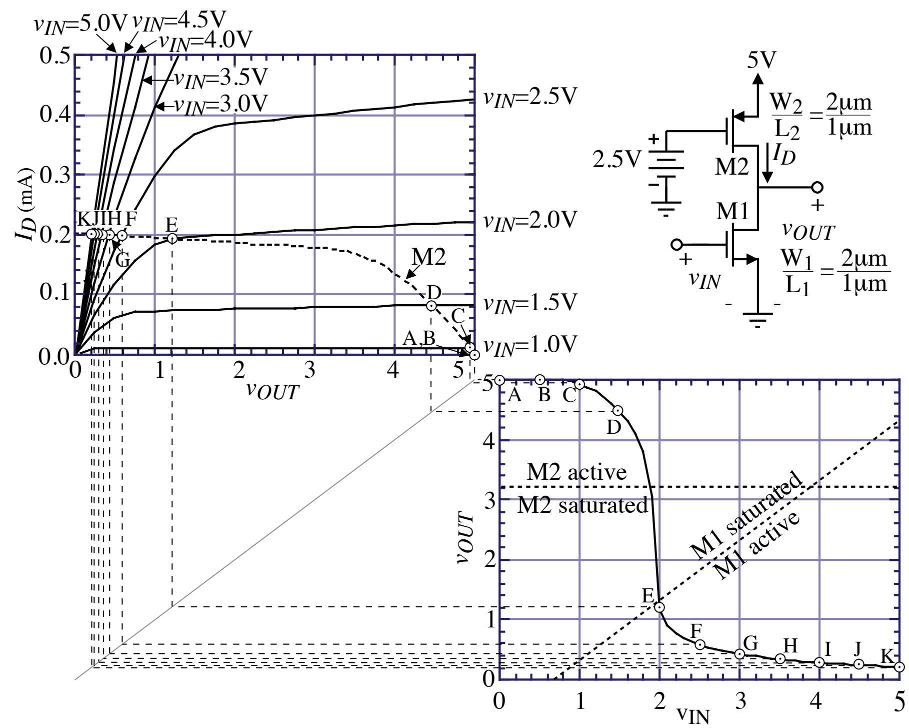


Fig. 5.1-5

Swing limits:

$$v_{OUT}(\text{max}) \cong V_{DD}$$

$$v_{OUT}(\text{min}) = (V_{DD} - V_{T1})\left[1 - \sqrt{1 - \left(\frac{\beta_2}{\beta_1}\right)\left(\frac{V_{DD} - V_{GG} - |V_{T2}|}{V_{DD} - V_{T1}}\right)^2}\,\right]$$

Small-Signal Midband Performance of the Current Source Load Inverter

Small-Signal Model:

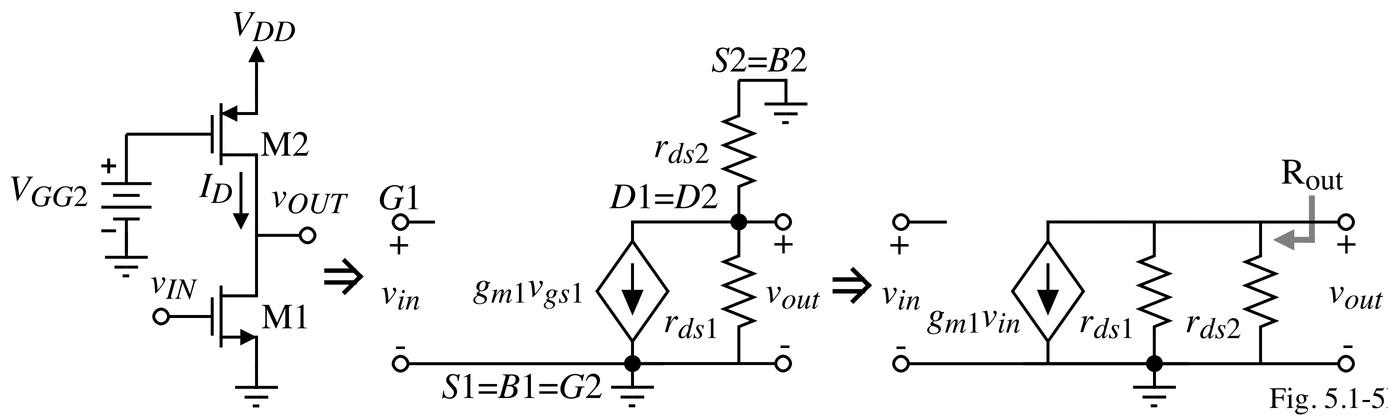
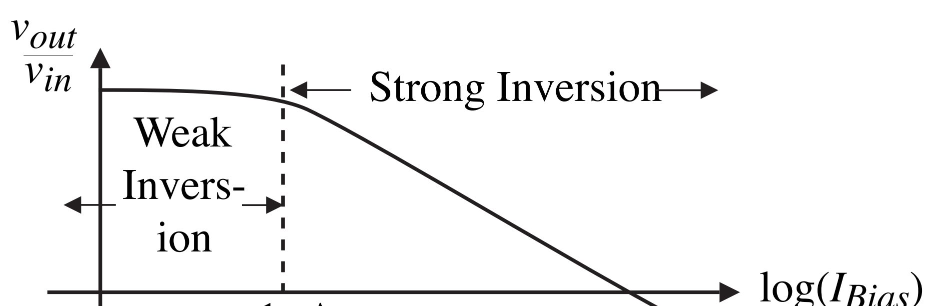


Fig. 5.1-5B

Midband Performance:

$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1}}{g_{ds1} + g_{ds2}} = \left(\frac{2K'_N W_1}{L_1 I_D}\right)^{1/2} \left(\frac{-1}{\lambda_1 + \lambda_2}\right) \propto \frac{1}{\sqrt{I_D}} \quad !!! \quad \text{and } R_{out} = \frac{1}{g_{ds1} + g_{ds2}} \cong \frac{1}{I_D(\lambda_1 + \lambda_2)}$$



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Frequency Response of the Current Source Load Inverter

Incorporation of the parasitic capacitors into the small-signal model (x is connected to V_{GG2}):

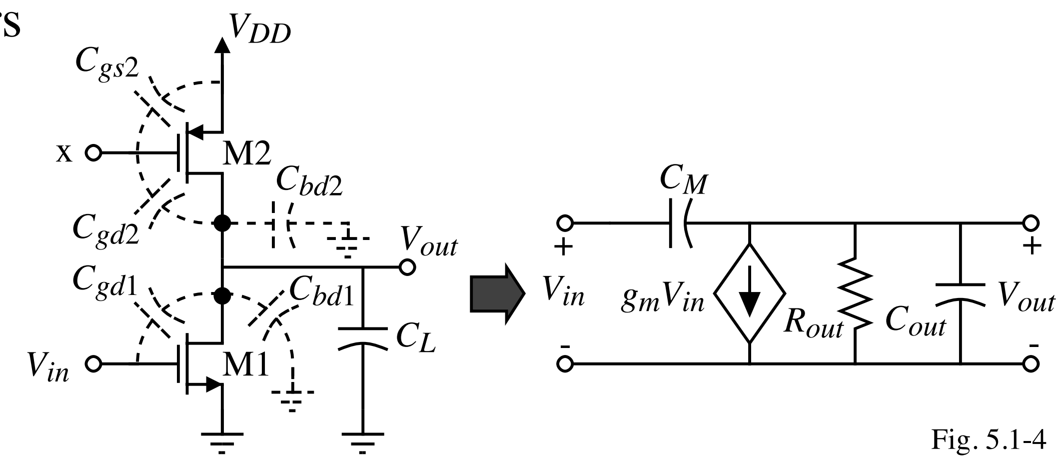


Fig. 5.1-4

If we assume the input voltage has a small source resistance, then we can write the following:

$$\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{-g_m R_{out}\left(1 - \frac{s}{z_1}\right)}{1 - \frac{s}{p_1}}$$

where $g_m = g_{m1}$, $p_1 = \frac{-1}{R_{out}(C_{out}+C_M)}$, and $z_1 = \frac{g_m}{C_M}$

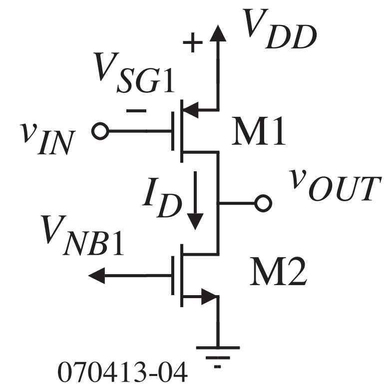
and $R_{out} = \frac{1}{g_{ds1} + g_{ds2}}$ and $C_{out} = C_{gd2} + C_{bd1} + C_{bd2} + C_L$ $C_M = C_{gd1}$

Therefore, if $|p_1|<|z_1|$, then the –3 dB frequency response can be expressed as

$$\omega_{\text{-3dB}} \approx \omega_1 = \frac{g_{ds1} + g_{ds2}}{C_{gd1} + C_{gd2} + C_{bd1} + C_{bd2} + C_L}$$

Example 18-2 - Performance of a Current-Sink Inverter

A current-sink inverter is shown. Assume that W_1 = 2 μm, L_1 = 1 μm, W_2 = 1 μm, L_2 = 1μm, V_{DD} = 5 volts, V_{NB1} = 3 volts, and the parameters of Table 3.1-2 describe M1 and M2. Use the capacitor values of Example 18-1 (C_{gd1} = C_{gd2}). Calculate the output-swing limits and the small-signal performance.



Solution

To attain the output signal-swing limitations, treat the current sink inverter as a current source CMOS inverter with PMOS (NMOS) parameters for the NMOS (PMOS) and use NMOS equations. Using a prime notation to designate the results of the current source CMOS inverter that exchanges the PMOS and NMOS model parameters,

$$v_{OUT}(\text{max})' = 5\text{V and } v_{OUT}(\text{min})' = (5\text{-}0.7)\left[1 - \sqrt{1 - \left(\frac{110\cdot 1}{50\cdot 2}\right)\left(\frac{3\text{-}0.7}{5\text{-}0\text{-}0.7}\right)^2}\,\right] = 0.74\text{V}$$

In terms of the current sink CMOS inverter, these limits are subtracted from 5V to get

$v_{OUT}(\text{max}) = 4.26\text{V}$ and $v_{OUT}(\text{min}) = 0\text{V}$.

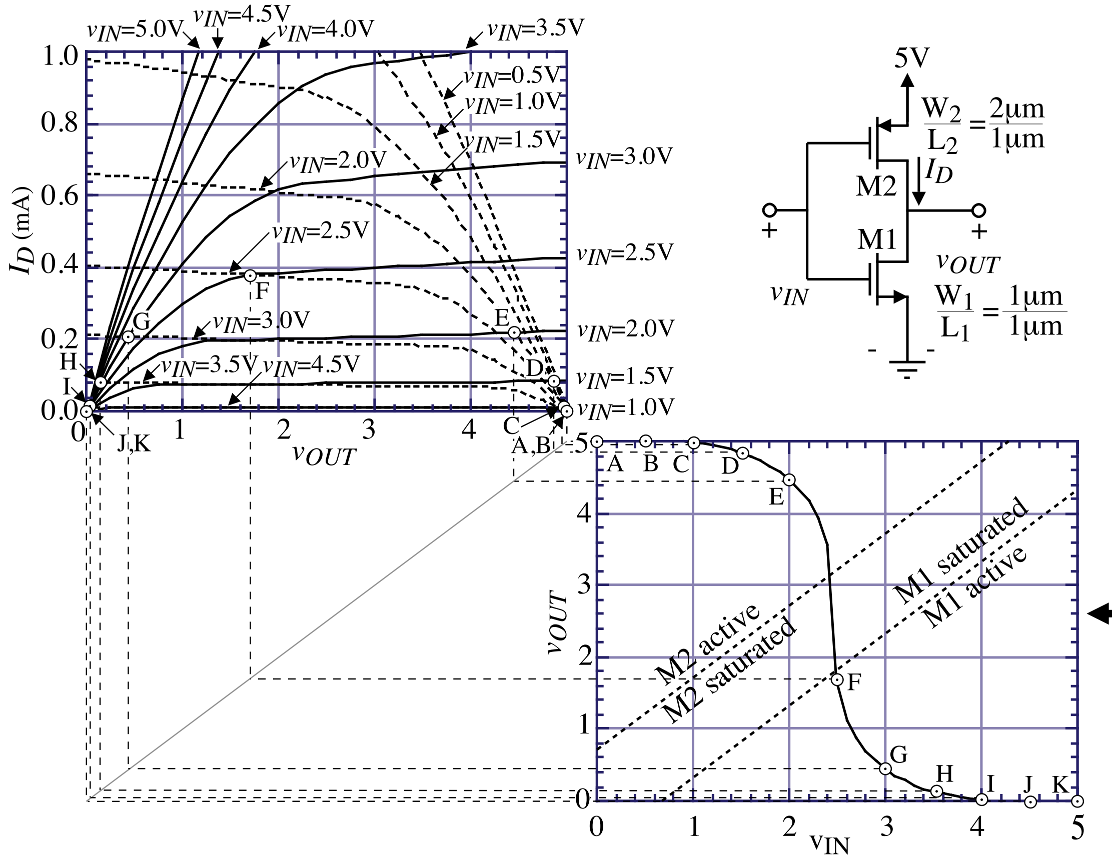
To find the small signal performance, first calculate the dc current. The dc current, I_D, is

$$I_D = \frac{K_N' W_1}{2L_1}(V_{GG1}\text{-}V_{TN})^2 = \frac{110\cdot 1}{2\cdot 1}(3\text{-}0.7)^2 = 291\mu\text{A}$$

v_{out}/v_{in} = –9.2V/V, R_{out} = 38.1 kΩ, and $f_{-3\text{dB}}$ = 2.78 MHz.

PUSH-PULL INVERTING AMPLIFIER

Voltage Transfer Characteristic of the Push-Pull Inverting Amplifier



Note the rail-to-rail output voltage swing

Fig. 5.1-8

Regions of operation for M1 and M2:

M1: $v_{DS1} \geq v_{GS1} - V_{T1} \rightarrow \quad v_{OUT} \geq v_{IN} - 0.7\text{V}$

M2: $v_{SD2} \geq v_{SG2} - |V_{T2}| \rightarrow \quad V_{DD} - v_{OUT} \geq V_{DD} - v_{IN} - |V_{T2}| \rightarrow \quad v_{OUT} \leq v_{IN} + 0.7\text{V}$

Small-Signal Performance of the Push-Pull Amplifier

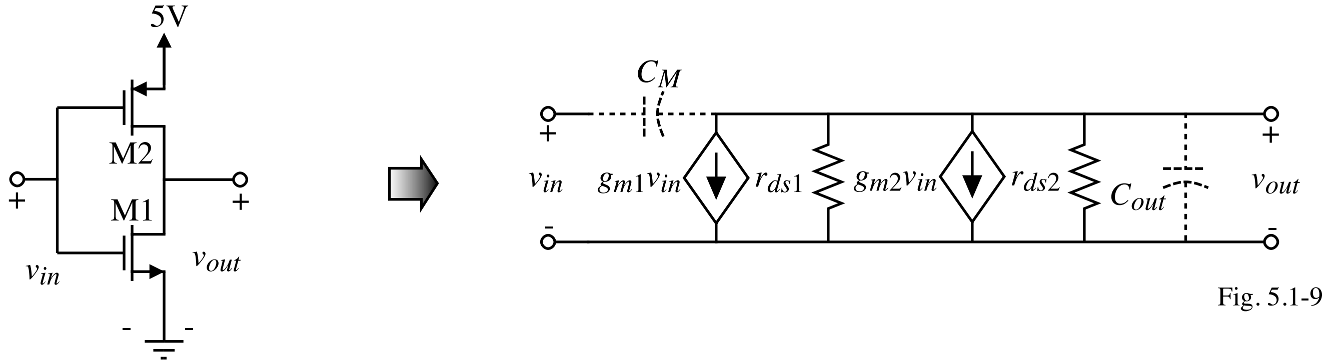


Fig. 5.1-9

Small-signal analysis gives the following results:

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{-(g_{m1} + g_{m2})}{g_{ds1} + g_{ds2}} = -\sqrt{(2/I_D)}\left[\frac{\sqrt{K'_N(W_1/L_1)} + \sqrt{K'_P(W_2/L_2)}}{\lambda_1 + \lambda_2}\right]$$

$$R_{out} = \frac{1}{g_{ds1} + g_{ds2}}$$

$$z = \frac{g_{m1}+g_{m2}}{C_M} = \frac{g_{m1}+g_{m2}}{C_{gd1}+C_{gd2}} \quad \text{and} \quad p_1 = \frac{-(g_{ds1} + g_{ds2})}{C_{gd1} + C_{gd2} + C_{bd1} + C_{bd2} + C_L}$$

If $z_1 > |p_1|$, then

$$\omega_{\text{-3dB}} = \frac{g_{ds1} + g_{ds2}}{C_{gd1} + C_{gd2} + C_{bd1} + C_{bd2} + C_L}$$

Example 18-3 - Performance of a Push-Pull Inverter

The performance of a push-pull CMOS inverter is to be examined. Assume that $W_1 = 1\ \mu\text{m}$, $L_1 = 1\ \mu\text{m}$, $W_2 = 2\ \mu\text{m}$, $L_2 = 1\mu\text{m}$, $V_{DD} = 5$ volts, and use the parameters of Table 3.1-2 to model M1 and M2. Use the capacitor values of Example 18-1 ($C_{gd1} = C_{gd2}$). Calculate the output-swing limits and the small-signal performance assuming that $I_{D1} = I_{D2} = 300\mu\text{A}$.

Solution

The output swing is seen to be from 0V to 5V. In order to find the small signal performance, we will make the important assumption that both transistors are operating in the saturation region. Therefore:

$$\frac{v_{out}}{v_{in}} = \frac{-257\mu\text{S} - 245\mu\text{S}}{12\mu\text{S} + 15\mu\text{S}} = -18.6\text{V/V}$$

$$R_{out} = 37\ \text{k}\Omega$$

$$f_{-3\text{dB}} = 2.86\ \text{MHz}$$

and

$$z_1 = 399\ \text{MHz}$$

NOISE ANALYSIS OF INVERTING AMPLIFIERS

Noise Analysis of Inverting Amplifiers

Noise model:

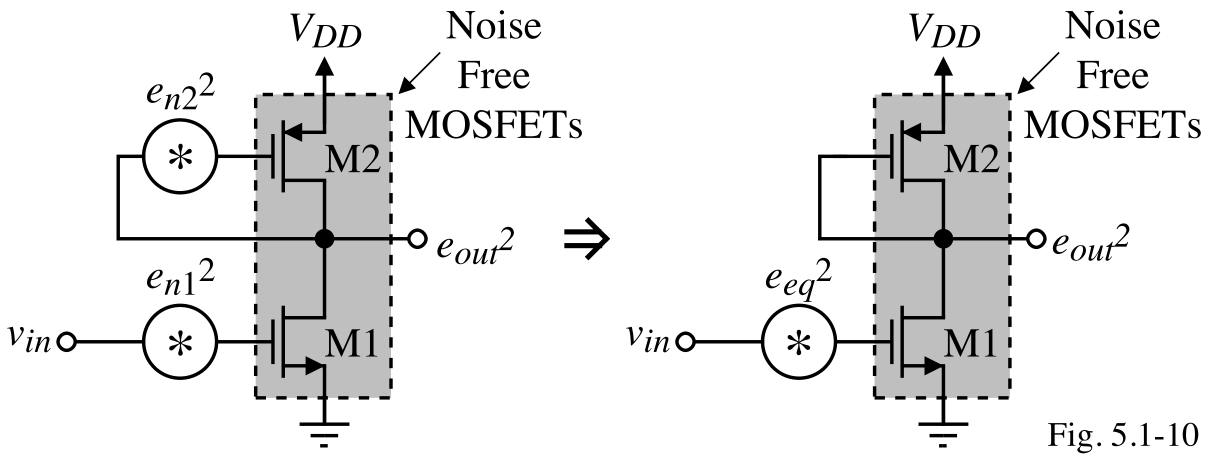


Fig. 5.1-10

Approach:

1.) Assume a mean-square input-voltage-noise spectral density e_n^2 in series with the gate of each MOSFET.
(This step assumes that the MOSFET is the common source configuration.)

2.) Calculate the output-voltage-noise spectral density, e_{out}^2 (Assume all sources are additive).

3.) Refer the output-voltage-noise spectral density back to the input to get equivalent input noise e_{eq}^2.

4.) Substitute the type of noise source, 1/f or thermal.

Noise Analysis of the Active Load Inverter

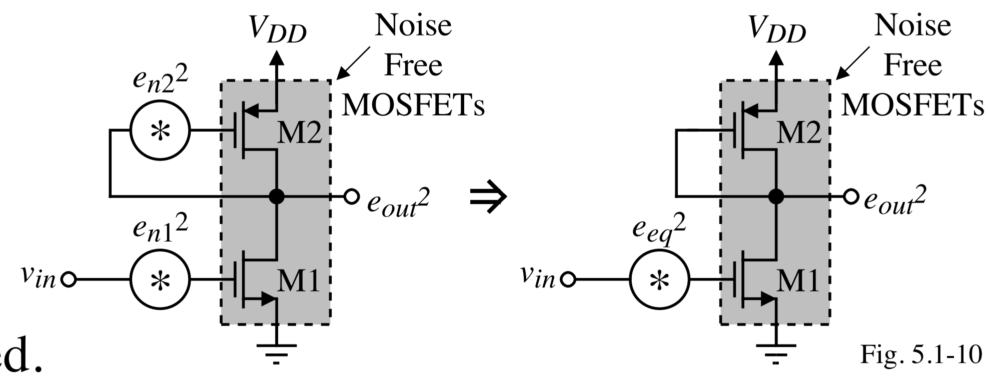


Fig. 5.1-10

1.) See model to the right.

2.) $e_{out}^2 = e_{n1}^2 \left(\frac{g_{m1}}{g_{m2}}\right)^2 + e_{n2}^2$

3.) $e_{eq}^2 = e_{n1}^2 \left[1 + \left(\frac{g_{m2}}{g_{m1}}\right)^2 \left(\frac{e_{n2}}{e_{n1}}\right)^2\right]$

Up to now, the type of noise is not defined.

1/f Noise

Substituting $e_n^2 = \frac{KF}{2fC_{ox}WLK'} = \frac{B}{fWL}$, into the above gives,

$$e_{eq(1/f)}^2 = \left(\frac{B_1}{fW_1L_1}\right)\left[1 + \left(\frac{K'_2B_2}{K'_1B_1}\right)\left(\frac{L_1}{L_2}\right)^2\right] \rightarrow \quad e_{eq(1/f)} = \left(\frac{B_1}{fW_1L_1}\right)^{1/2}\left[1 + \left(\frac{K'_2B_2}{K'_1B_1}\right)\left(\frac{L_1}{L_2}\right)^2\right]^{1/2}$$

To minimize 1/f noise, 1.) Make $L_2 >> L_1$, 2.) Increase W_1 and 3.) choose M1 as a PMOS.

Thermal Noise

Substituting $e_n^2 = \frac{8kT}{3g_m}$ into the above gives,

$$e_{eq(th)} = \left\{\left(\frac{8kT}{3[2K'_1(W/L)_1I_1]^{1/2}}\right)\left[1 + \left(\frac{W_2L_1K'_2}{L_2W_1K'_1}\right)^{1/2}\right]\right\}^{1/2}$$

To minimize thermal noise, maximize the gain of the inverter.

Noise Analysis for Weak Inversion

How does the analysis change for weak inversion operation?

Small signal transconductance is $g_m = \dfrac{I_D}{nV_t} = \dfrac{qI_D}{nkT}$

Noise sources in weak inversion:

1) $1/f$ noise given as $e_n^2 = \dfrac{\text{KF}}{2fC_{ox}WLK'} = \dfrac{B}{fWL}$

$$e_{eq}(1/f)^2 = e_{n1}^2\left[1+\left(\frac{g_{m2}}{g_{m1}}\right)^2\left(\frac{e_{n2}}{e_{n1}}\right)^2\right] = \left(\frac{B_1}{fW_1L_1}\right)\left[1+\left(\frac{I_{D2}/n_2V_t}{I_{D1}/n_1V_t}\right)^2\left(\frac{B_2/f\,W_2L_2}{B_1/f\,W_1L_1}\right)\right]$$

$$= \left(\frac{B_1}{fW_1L_1}\right)\left[1+\left(\frac{n_1{}^2B_2W_1L_1}{n_2{}^2B_1W_2L_2}\right)\right]$$

2.) Thermal noise given as $e_n^2 = \dfrac{8kT}{3g_m}$

$$e_{eq}(\text{th})^2 = e_{n1}^2\left[1+\left(\frac{g_{m2}}{g_{m1}}\right)^2\left(\frac{g_{m1}}{g_{m2}}\right)\right] = \left(\frac{8kT}{3g_{m1}}\right)\left[1+\left(\frac{g_{m2}}{g_{m1}}\right)\right] = \left(\frac{8kT}{3g_{m1}}\right)\left[1+\left(\frac{n_1}{n_2}\right)\right]$$

Therefore, weak inversion operation does not lend itself to easy minimization of the $1/f$ or thermal noise.

Noise Analysis of the Current Source Load Inverting Amplifier

Model:

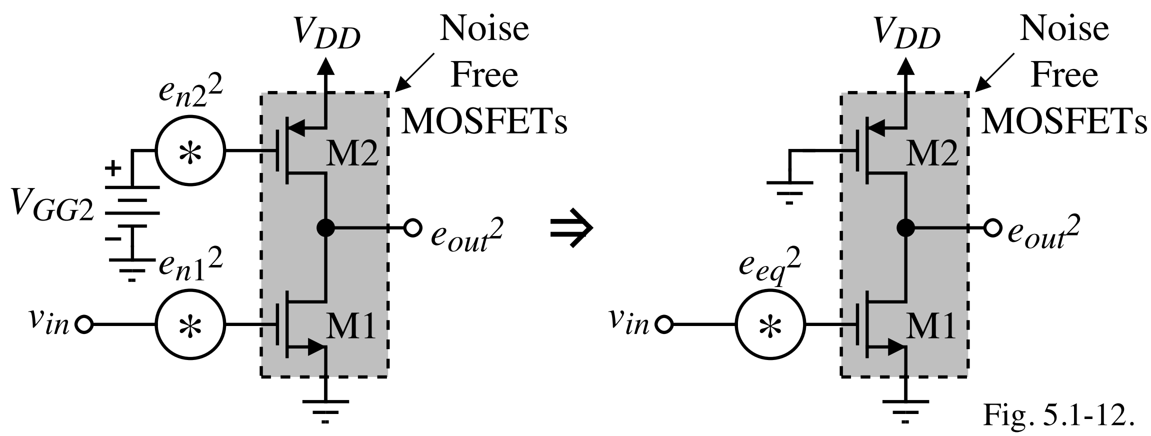


Fig. 5.1-12.

The output-voltage-noise spectral density of this inverter can be written as,

$$e_{out}^2 = (g_{m1}r_{out})^2 e_{n1}^2 + (g_{m2}r_{out})^2 e_{n2}^2$$

or

$$e_{eq}^2 = e_{n1}^2 + \frac{(g_{m2}r_{out})^2}{(g_{m1}r_{out})^2} e_{n2}^2 = e_{n1}^2 \left[1 + \left(\frac{g_{m2}}{g_{m1}}\right)^2 \frac{e_{n2}^2}{e_{n1}^2}\right]$$

This result is identical with the active load inverter.

Thus the noise performance of the two circuits are equivalent although the small-signal voltage gain is significantly different.

Noise Analysis of the Push-Pull Amplifier

Model:

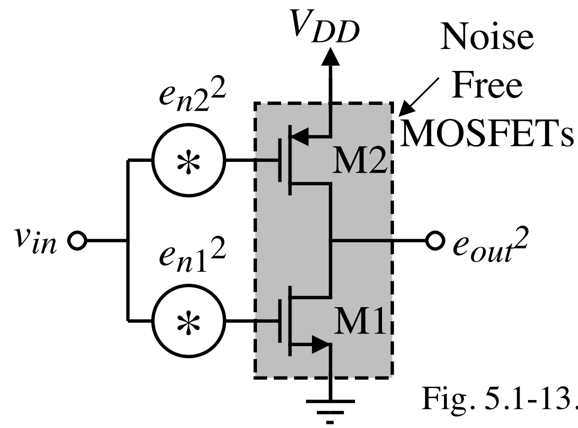


Fig. 5.1-13.

The equivalent input-voltage-noise spectral density of the push-pull inverter can be found as

$$e_{eq} = \sqrt{\left(\frac{g_{m1}e_{n1}}{g_{m1}+g_{m2}}\right)^2 + \left(\frac{g_{m2}e_{n2}}{g_{m1}+g_{m2}}\right)^2}$$

If the two transconductances are balanced ($g_{m1} = g_{m2}$), then the noise contribution of each device is divided by two.

The total noise contribution can only be reduced by reducing the noise contribution of each device.

(Basically, both M1 and M2 act like the "load" transistor and "input" transistor, so there is no defined input transistor that can cause the noise of the load transistor to be insignificant.)

SUMMARY

Table of Performance

Inverter	AC Voltage Gain	AC Output Resistance	Bandwidth (CGB=0)	Equivalent, input-referred, mean-square noise voltage
p-channel active load inverter	$\frac{-g_{m1}}{g_{m2}}$	$\frac{1}{g_{m2}}$	$\frac{g_{m2}}{C_{BD1}+C_{GS1}+C_{GS2}+C_{BD2}}$	$e_{n1}^2 + e_{n2}^2\left(\frac{g_{m2}}{g_{m1}}\right)^2$
Current source load inverter	$\frac{-g_{m1}}{g_{ds1}+g_{ds2}}$	$\frac{1}{g_{ds1}+g_{ds2}}$	$\frac{g_{ds1}+g_{ds2}}{C_{BD1}+C_{GD1}+C_{DG2}+C_{BD2}}$	$e_{n1}^2 + e_{n2}^2\left(\frac{g_{m2}}{g_{m1}}\right)^2$
Push-Pull inverter	$\frac{-(g_{m1}+g_{m2})}{g_{ds1}+g_{ds2}}$	$\frac{1}{g_{ds1}+g_{ds2}}$	$\frac{g_{ds1}+g_{ds2}}{C_{BD1}+C_{GD1}+C_{GS2}+C_{BD2}}$	$\left(\frac{g_{m1}e_{n1}}{g_{m1}+g_{m2}}\right)^2+\left(\frac{g_{m1}e_{n1}}{g_{m1}+g_{m2}}\right)^2$