LECTURE 160 – CURRENT MIRRORS AND SIMPLE REFERENCES

LECTURE ORGANIZATION

Outline
• MOSFET current mirrors
• Improved current mirrors
• Voltage references with power supply independence
• Current references with power supply independence
• Temperature behavior of voltage and current references

CMOS Analog Circuit Design, 2nd Edition Reference
Pages 134-153

MOSFET CURRENT MIRRORS

What is a Current Mirror?
A current mirror replicates the input current of a current sink or current source as an output current. The output current may be identical to the input current or can be a scaled version of it.

The above current mirrors are referenced with respect to ground. Current mirrors can also be referenced with respect to $V_{DD}$ and current sink inputs and outputs.
Characterization of Current Mirrors

A current mirror is basically nothing more than a current amplifier. The ideal characteristics of a current amplifier are:

- Output current linearly related to the input current, \( i_{\text{out}} = A_i i_{\text{in}} \)
- Input resistance is zero
- Output resistance is infinity

Also, the characteristic \( V_{\text{MIN}} \) applies not only to the output but also the input.

- \( V_{\text{MIN}}(\text{in}) \) is the range of \( v_{\text{in}} \) over which the input resistance is not small
- \( V_{\text{MIN}}(\text{out}) \) is the range of \( v_{\text{out}} \) over which the output resistance is not large

Graphically:

Therefore, \( R_{\text{out}}, R_{\text{in}}, V_{\text{MIN}}(\text{out}), V_{\text{MIN}}(\text{in}) \), and \( A_i \) will characterize the current mirror.

Simple MOS Current Mirror

Circuit:

Assume that \( v_{DS2} > v_{GS} - V_{T2} \), then
\[
\frac{i_O}{i_I} = \frac{L_1 W_2}{W_1 L_2} \left( \frac{V_{GS} - V_{T2}}{V_{GS} - V_{T1}} \right) \left( \frac{1 + \lambda v_{DS2} K_2'}{1 + \lambda v_{DS1} K_1'} \right)
\]

If the transistors are matched, then \( K_1' = K_2' \) and \( V_{T1} = V_{T2} \) to give,
\[
\frac{i_O}{i_I} = \frac{L_1 W_2}{W_1 L_2} \left( 1 + \lambda v_{DS2} \right)
\]

If \( v_{DS1} = v_{DS2} \), then
\[
\frac{i_O}{i_I} = \frac{L_1 W_2}{W_1 L_2}
\]

Therefore the sources of error are:

1.) \( v_{DS1} \approx v_{DS2} \)
2.) M1 and M2 are not matched.
Influence of the Channel Modulation Parameter, $\lambda$

If the transistors are matched and the W/L ratios are equal, then

$$\frac{i_O}{i_I} = \frac{1 + \lambda \Delta V_{DS2}}{1 + \lambda \Delta V_{DS1}}$$

if the channel modulation parameter is the same for both transistors ($L_1 = L_2$).

Ratio error (%) versus drain voltage difference:

Note that one could use this effect to measure $\lambda$.

Measure $V_{DS1}, V_{DS2}, i_I$ and $i_O$ and solve the above equation for the channel modulation parameter, $\lambda$.

Illustration of the Offset Voltage Error Influence

Assume that $V_{T1} = 0.7\text{V}$ and $K'W/L = 110\mu\text{A/V}^2$.

Key: Make the part of $V_{GS}$ causing the current to flow, $V_{ON}$, more significant than $V_T$. 
Influence of Error in Aspect Ratio of the Transistors

Example 160-1 - Aspect Ratio Errors in Current Mirrors

A layout is shown for a one-to-four current amplifier. Assume that the lengths are identical \((L_1 = L_2)\) and find the ratio error if \(W_1 = 5 \pm 0.1 \mu m\). The actual widths of the two transistors are

\[
W_1 = 5 \pm 0.1 \mu m \quad \text{and} \quad W_2 = 20 \pm 0.1 \mu m
\]

Solution

We note that the tolerance is not multiplied by the nominal gain factor of 4. The ratio of \(W_2\) to \(W_1\) and consequently the gain of the current amplifier is

\[
\frac{i_O}{i_I} = \frac{W_2}{W_1} = \frac{20 \pm 0.1}{5 \pm 0.1} = 4 \left( 1 \pm \frac{0.1}{20} \right) = 4 \left( 1 \pm \frac{0.1}{5} \right) = 4 \left( 1 \pm \frac{0.4}{20} \right) = 4 \pm (0.03)
\]

where we have assumed that the variations would both have the same sign (correlated). It is seen that this ratio error is 0.75% of the desired current ratio or gain.

Example 160-2 - Reduction of the Aspect Ratio Errors in Current Mirrors

Use the layout technique illustrated below and calculate the ratio error of a current amplifier having the specifications of the previous example.

Solutions

The actual widths of M1 and M2 are

\[
W_1 = 5 \pm 0.1 \mu m \quad \text{and} \quad W_2 = 4(5 \pm 0.1) \mu m
\]

The ratio of \(W_2\) to \(W_1\) and consequently the current gain is given below and is for all practical purposes independent of layout error.

\[
\frac{i_O}{i_I} = \frac{4(5 \pm 0.1)}{5 \pm 0.1} = 4
\]
Summary of the Simple MOS Current Mirror/Amplifier

- Minimum input voltage is $V_{\text{MIN}(\text{in})} = V_T + V_{\text{ON}}$
  
  Okay, but could be reduced to $V_{\text{ON}}$.
  
  Principle:

$\begin{align*}
\text{Fig. 300-7}
\end{align*}$

Will deal with later in low voltage op amps.

- Minimum output voltage is $V_{\text{MIN}(\text{out})} = V_{\text{ON}}$

- Output resistance is $R_{\text{out}} = \frac{1}{g_m}$

- Input resistance is $R_{\text{in}} = \frac{1}{g_m}$

- Current gain accuracy is poor because $v_{DS1} \neq v_{DS2}$

IMPROVED CURRENT MIRRORS

Large Output Swing Cascode Current Mirror

- $R_{\text{out}} = g_m2r_{ds2}r_{ds1}$

- $R_{\text{in}} = ?$ $v_{in} = r_{ds5}(i_{in} - g_m5v_{g5}) + v_{s5} = r_{ds5}(i_{in} + g_m5v_{g5}) + v_{s5} = r_{ds5}i_{in} + (1+g_m5r_{ds5})v_{s5}$

  But, $v_{s5} \neq r_{ds3}(i_{in} - g_m3v_{g3})$

  $\therefore v_{in} = r_{ds5}i_{in} + (1+g_m5r_{ds5})r_{ds3}i_{in} - g_m3r_{ds3}(1+g_m5r_{ds5})v_{in}$

  $R_{\text{in}} = \frac{v_{in}}{i_{in}} = \frac{r_{ds5} + r_{ds3} + r_{ds3}g_m5r_{ds5}}{g_m3r_{ds3}(1+g_m5r_{ds5})} \approx \frac{1}{g_m3}$

- $V_{\text{MIN}(\text{out})} = 2V_{\text{ON}}$

- $V_{\text{MIN}(\text{in})} = V_T + V_{\text{ON}}$

- Current gain is excellent because $v_{DS1} = v_{DS3}$. 
Self-Biased Cascode Current Mirror

- $R_{in} =$ ?
- $v_{in} = i_{in}R + r_{ds3}(i_{in} - g_{m3}v_{gs3}) + r_{ds1}(i_{in} - g_{m1}v_{gs1})$

But,
- $v_{gs1} = v_{in} - i_{in}R$

and
- $v_{gs3} = v_{in} - r_{ds1}(i_{in} - g_{m1}v_{gs1})$
- $= v_{in} - r_{ds1}i_{in} + g_{m1}r_{ds1}(v_{in} - i_{in}R)$

Thus,
- $v_{in} = i_{in}R + r_{ds3}i_{in} - g_{m3}r_{ds3}[v_{in} - r_{ds1}i_{in} + g_{m1}r_{ds1}(v_{in} - i_{in}R)] + r_{ds1}[i_{in} - g_{m1}(v_{in} + i_{in}R)]$
- $= i_{in}[R + r_{ds1} + r_{ds3} + g_{m3}r_{ds3}r_{ds1} + g_{m1}r_{ds1} + g_{m1}r_{ds18} + g_{m1}r_{ds3} + g_{m1}r_{ds3}r_{ds3} + g_{m1}r_{ds3}r_{ds3} + g_{m1}r_{ds3}r_{ds3}]$
- $R_{in} = \frac{1}{1 + g_{m3}r_{ds3} + g_{m1}r_{ds18} + g_{m1}r_{ds3}r_{ds3} + g_{m1}r_{ds3}r_{ds3} + g_{m1}r_{ds3}r_{ds3}} \approx \frac{1}{g_{m1} + R}$

- $R_{out} \approx g_{m4}r_{ds4}r_{ds2}$
- $V_{MIN}(in) = V_T + 2V_{ON}$
- $V_{MIN}(out) = 2V_{ON}$
- Current gain matching is excellent

MOS Regulated Cascode Current Mirror

- $R_{out} \approx g_{m2}r_{ds3}$
- $R_{in} \approx \frac{1}{g_{m4}}$
- $V_{MIN}(out) = V_T + 2V_{ON}$  (Can be reduced to $2V_{ON}$)
- $V_{MIN}(in) = V_T + V_{ON}$  (Can be reduced to $V_{ON}$)
- Current gain matching - good as long as $v_{DS4} = v_{DS2}$
### Summary of MOS Current Mirrors

<table>
<thead>
<tr>
<th>Current Mirror</th>
<th>Accuracy</th>
<th>Output Resistance</th>
<th>Input Resistance</th>
<th>Minimum Output Voltage</th>
<th>Minimum Input Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>Poor</td>
<td>$r_{ds}$</td>
<td>$\frac{1}{g_m}$</td>
<td>$V_{ON}$</td>
<td>$V_T + V_{ON}$</td>
</tr>
<tr>
<td>Wide Output Swing Cascode</td>
<td>Excellent</td>
<td>$g_{m}r_{ds}^2$</td>
<td>$\frac{1}{g_m}$</td>
<td>$2V_{ON}$</td>
<td>$V_T + V_{ON}$</td>
</tr>
<tr>
<td>Self-biased Cascode</td>
<td>Excellent</td>
<td>$g_{m}r_{ds}^2$</td>
<td>$R + \frac{1}{g_m}$</td>
<td>$2V_{ON}$</td>
<td>$V_T + 2V_{ON}$</td>
</tr>
<tr>
<td>Regulated Cascode</td>
<td>Good-Excellent</td>
<td>$g_{m}^2r_{ds}^3$</td>
<td>$\frac{1}{g_m}$</td>
<td>$V_T + 2V_{ON}$ (Can be $2V_{ON}$)</td>
<td>$V_T + V_{ON}$ (Can be $\approx V_{ON}$)</td>
</tr>
</tbody>
</table>

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**Voltage References with Power Supply Independence**

**Power Supply Independence**

How do you characterize power supply independence?

Use the concept of:

$$ S_{V_{DD}}^{V_{REF}} = \frac{\partial V_{REF}/V_{REF}}{\partial V_{DD}/V_{DD}} = \frac{V_{DD}}{V_{REF}} \left( \frac{\partial V_{REF}}{\partial V_{DD}} \right) $$

Application of sensitivity to determining power supply dependence:

$$ \frac{\partial V_{REF}}{V_{REF}} = \left( S_{V_{DD}}^{V_{REF}} \right) \frac{\partial V_{DD}}{V_{DD}} $$

Thus, the fractional change in the reference voltage is equal to the sensitivity times the fractional change in the power supply voltage.

For example, if the sensitivity is 1, then a 10% change in $V_{DD}$ will cause a 10% change in $V_{REF}$.

Ideally, we want $S_{V_{DD}}^{V_{REF}}$ to be zero for power supply independence.
MOSFET-Resistance Voltage References

\[ V_{REF} = V_{GS} = V_T + \sqrt{\frac{2(V_{DD} - V_{REF})}{\beta R}} \]

or

\[ V_{REF} = V_T - \frac{V_T}{\beta R} + \sqrt{\frac{2(V_{DD} - V_T)^2}{\beta R} + \frac{1}{(\beta R)^2}} \]

\[ S_{V_{DD}} = \left( \frac{1}{\sqrt{1 + 2\beta(V_{DD} - V_T)R}} \right) \frac{V_{DD}}{V_{REF}} \]

Assume that \( V_{DD}=5V \), \( W/L =100 \) and \( R=100k\Omega \),

Thus, \( V_{REF} \approx 0.7875V \) and \( S_{V_{DD}} = 0.0653 \)

Bipolar-Resistance Voltage References

\[ V_{REF} = V_{EB} = \frac{kT}{q} \ln \left( \frac{I}{I_s} \right) \]

and \( I = \frac{V_{CC} - V_{EB}}{R} = \frac{V_{CC}}{R} \)

give \( V_{REF} = \frac{kT}{q} \ln \left( \frac{V_{CC}}{R I_s} \right) \)

\[ S_{V_{CC}} = \frac{1}{\ln[V_{CC}/(RI_s)]} = \frac{1}{\ln(I/I_s)} \]

If \( V_{CC} = 5V \), \( R = 4.3k\Omega \) and \( I_s = 1fA \),

then \( V_{REF} = 0.719V \).

Also, \( S_{V_{CC}} = 0.0362 \)

If the current in \( R_1 \) (and \( R_2 \)) is small compared to the current flowing through the transistor, then

\[ V_{REF} = \left( \frac{R_1 + R_2}{R_1} \right) V_{EB} \]

Can use diodes in place of the BJTs.
CURRENT REFERENCES WITH POWER SUPPLY INDEPENDENCE

Power Supply Independence

Again, we want
\[ \frac{I_{REF}}{V_{DD}} = \frac{\partial I_{REF}}{\partial V_{DD}} = \frac{V_{DD}}{I_{REF}} \left( \frac{\partial I_{REF}}{\partial V_{DD}} \right) \]
to approach zero.

Therefore, as \( \frac{I_{REF}}{V_{DD}} \) approaches zero, the change in \( I_{REF} \) as a function of a change in \( V_{DL} \) approaches zero.

Gate-Source Referenced Current Reference

The circuit below uses both positive and negative feedback to accomplish a current reference that is reasonably independent of power supply.

Circuit:

Principle:

If \( M3 = M4 \), then \( I_1 = I_2 \). However, the M1-R loop gives
\[ V_{GS1} = V_{T1} + \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}} \]

Solving these two equations gives
\[ I_2 = \frac{V_{GS1}}{R} = \frac{V_{T1}}{R} + \left( \frac{1}{R} \right) \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}} \]

The output current, \( I_{out} = I_1 = I_2 \) can be solved as
\[ I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R^2} + \frac{1}{(\beta_1 R)^2}} \]
Simulation Results for the Gate-Source Referenced Current Reference

The current $I_D$ appears to be okay, why is $I_D$ increasing? Apparently, the channel modulation on the current mirror M3-M4 is large.

At $V_{DD} = 5V$, $V_{SD3} = 2.83V$ and $V_{SD4} = 1.09V$ which gives $I_D = 1.067I_D = 107\mu A$

Need to cascode the upper current mirror.

SPICE Input File:

Simple, Bootstrap Current Reference
VDD 1 0 DC 5.0
VSS 9 0 DC 0.0
M1 5 7 9 N W=20U L=1U
M2 3 5 7 N W=20U L=1U
M3 5 3 1 P W=25U L=1U
M4 3 3 1 P W=25U L=1U
M5 9 3 1 P W=25U L=1U
R 7 9 10KILOHM
M8 6 6 9 N W=1U L=1U
M7 6 6 5 9 N W=20U L=1U

Cascoded Gate-Source Referenced Current Reference

SPICE Input File:

Cascode, Bootstrap Current Reference
VDD 1 0 DC 5.0
VSS 9 0 DC 0.0
M1 5 7 9 N W=20U L=1U
M2 4 5 7 9 N W=20U L=1U
M3 2 3 1 1 P W=25U L=1U
M4 3 3 1 1 P W=25U L=1U
M3C 5 4 2 1 P W=25U L=1U
MC4 3 4 8 1 P W=25U L=1U
RON 3 4 4KILOHM
M5 9 3 1 1 P W=25U L=1U
R 7 9 10KILOHM
M8 6 6 9 9 N W=1U L=1U

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**Base-Emitter Referenced Circuit**

\[
I_{out} = I_2 = \frac{V_{EB1}}{R}
\]

BJT can be a MOSFET in weak inversion.

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**Low Voltage Gate-Source Referenced MOS Current Reference**

The previous gate-source referenced circuits required at least 2 volts across the power supply before operating.

A low-voltage gate-source referenced circuit:

Without the batteries, \( V_T \), the minimum power supply is \( V_T + 2V_{ON} + V_R \).

With the batteries, \( V_T \), the minimum power supply is \( 2V_{ON} + V_R \approx 0.5V \).
Summary of Power-Supply Independent References

- Reasonably good, simple voltage and current references are possible
- Best power supply sensitivity is approximately 0.01
  (10% change in power supply causes a 0.1% change in reference)

<table>
<thead>
<tr>
<th>Type of Reference</th>
<th>$V_{\text{REF}}$</th>
<th>$I_{\text{REF}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage division</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simple Current Reference</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>MOSFET-R</td>
<td>&lt;&lt;1</td>
<td></td>
</tr>
<tr>
<td>BJT-R</td>
<td>&lt;&lt;1</td>
<td></td>
</tr>
<tr>
<td>Gate-source Referenced</td>
<td>&lt;&lt;1</td>
<td></td>
</tr>
<tr>
<td>Base-emitter Referenced</td>
<td>&lt;&lt;1</td>
<td></td>
</tr>
</tbody>
</table>

TEMPERATURE BEHAVIOR OF VOLTAGE AND CURRENT REFERENCES

Characterization of Temperature Dependence

The objective is to minimize the fractional temperature coefficient defined as,

$$TC_F = \frac{1}{V_{\text{REF}}} \left( \frac{\partial V_{\text{REF}}}{\partial T} \right) = \frac{1}{T} \frac{V_{\text{REF}}}{S} \text{ parts per million per } ^\circ\text{C or ppm/}^\circ\text{C}$$

Temperature dependence of PN junctions:

$$i \approx I_s \exp \left( \frac{v}{V_T} \right)$$

$$I_s = KT^3 \exp \left( -\frac{V_{GO}}{V_T} \right)$$

$$\frac{dv_{BE}}{dT} = \frac{V_{BE} - V_{GO}}{T} = -2mV/^\circ\text{C at room temperature}$$

($V_{GO} = 1.205 \text{ V at room temperature and is called the bandgap voltage}$)

Temperature dependence of MOSFET in strong inversion:

$$\frac{dv_{GS}}{dT} = \frac{dV_T}{dT} + \sqrt{\frac{2L}{W C_{ox}}} \frac{d}{dT} \sqrt{\frac{i_D}{\mu_o}}$$

$$\mu_o = KT^{-1.5}$$

$$V_T(T) = V_T(T_0) - \alpha(T-T_0)$$

Resistors: \(\frac{1}{R}(dR/dT)\) ppm/°C
**Bipolar-Resistance Voltage References**

From previous work we know that,

\[
V_{\text{REF}} = \frac{kT}{q} \ln \left( \frac{V_{\text{DD}} - V_{\text{REF}}}{R I_s} \right)
\]

However, not only is \( V_{\text{REF}} \) a function of \( T \), but \( R \) and \( I_s \) are also functions of \( T \).

\[
\frac{dV_{\text{REF}}}{dT} = \frac{k}{q} \ln \left( \frac{V_{\text{DD}} - V_{\text{REF}}}{R I_s} \right) + \frac{kT}{q} \left( \frac{RI_s}{V_{\text{DD}} - V_{\text{REF}}} \right) \frac{dV_{\text{REF}}}{dT} - \frac{\left( V_{\text{DD}} - V_{\text{REF}} \right)}{R I_s} \left( \frac{dR}{dT} + \frac{dI_s}{dT} \right)
\]

\[
= \frac{V_{\text{REF}}}{T} - \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} - V_t \frac{dR}{R dT} + \frac{dI_s}{I_s dT} = \frac{V_{\text{REF}} - V_{\text{GO}}}{T} - \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} - \frac{3V_t}{T} \frac{dR}{R dT}
\]

\[
\therefore \frac{dV_{\text{REF}}}{dT} = \frac{V_{\text{REF}}}{T} \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} - \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dR}{R dT} - \frac{3V_t}{T}
\]

\[
TC_F = \frac{1}{V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} = \frac{V_{\text{REF}} - V_{\text{GO}}}{T} - \frac{V_t}{V_{\text{DD}} - V_{\text{REF}}} \frac{dR}{R dT} - \frac{3V_t}{T}
\]

If \( V_{\text{REF}} = 0.6V \), \( V_t = 0.026V \), and the \( R \) is polysilicon, then at 27°C the \( TC_F \) is

\[
TC_F = \frac{0.6-1.205}{0.6-300} \frac{V_{\text{REF}}}{V_{\text{DD}} - V_{\text{REF}}} = 33110-65x10^{-6}-433x10^{-6} = -3859\text{ppm/°C}
\]

**MOSFET Resistor Voltage Reference**

From previous results we know that

\[
V_{\text{REF}} = V_{\text{GS}} = V_{\text{T}} + \sqrt{\frac{2(V_{\text{DD}} - V_{\text{REF}})}{\beta R}}
\]

or

\[
V_{\text{REF}} = V_{\text{T}} - \frac{1}{\beta R} + \sqrt{\frac{2(V_{\text{DD}} - V_{\text{T}})}{\beta R}} + \frac{1}{(\beta R)^2}
\]

Note that \( V_{\text{REF}}, V_{\text{T}}, \beta, \) and \( R \) are all functions of temperature.

It can be shown that the \( TC_F \) of this reference is

\[
\frac{dV_{\text{REF}}}{dT} = -\alpha + \sqrt{\frac{V_{\text{DD}} - V_{\text{REF}}}{2\beta R} \left( \frac{1.5}{T} - 1 \frac{dR}{R dT} \right)}
\]

\[
= \frac{1}{1 + \sqrt{2\beta R (V_{\text{DD}} - V_{\text{REF}})}}
\]

\[
\therefore \frac{dV_{\text{REF}}}{dT} = \frac{1}{V_{\text{REF}}(1 + \sqrt{2\beta R (V_{\text{DD}} - V_{\text{REF}})})}
\]

\[
TC_F = \frac{1}{V_{\text{REF}}} \frac{dV_{\text{REF}}}{dT} = \frac{1}{V_{\text{REF}}(1 + \sqrt{2\beta R (V_{\text{DD}} - V_{\text{REF}})})}
\]
Example 160-3 - Calculation of MOSFET-Resistor Voltage Reference $TC_F$

Calculate the temperature coefficient of the MOSFET-Resistor voltage reference where $W/L=2$, $V_{DD}=5V$, $R=100k\Omega$ using the parameters of Table 3.1-2. The resistor, $R$, is polysilicon and has a temperature coefficient of 1500 ppm/$^\circ$C.

Solution

First, calculate $V_{REF}$. Note that $\beta R = 220\times10^{-6}\times10^5 = 22$ and $\frac{dR}{RdT} = 1500\text{ppm}/^\circ\text{C}$.

$V_{REF} = 0.7 \frac{1}{22} + \sqrt{\frac{2(5 - 0.7)}{22} + \left(\frac{1}{22}\right)^2} = 1.281V$

Now,

$\frac{dV_{REF}}{dT} = -2.3\times10^{-3} + \sqrt{\frac{5 - 1.281}{2(22)}} \left(\frac{1.5}{300} - 1500\times10^{-6}\right) \frac{1}{1 + \sqrt{2(22)}(5 - 1.281)} = -1.189\times10^{-3}\text{V}/^\circ\text{C}$

The fractional temperature coefficient is given by

$TC_F = -1.189\times10^{-3} \left(\frac{1}{1.281}\right) = -928\text{ ppm}/^\circ\text{C}$

Gate-Source and Base-Emitter Referenced Current Source/Sinks

Gate-source referenced source:

The output current was given as, $I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R} + \left(\frac{1}{\beta_1 R}\right)^2}$

Although we could grind out the derivative of $I_{out}$ with respect to $T$, the temperature performance of this circuit is not that good to spend the time to do so. Therefore, let us assume that $V_{GS1} = V_{T1}$ which gives

$I_{out} \approx \frac{V_{T1}}{R} \quad \Rightarrow \quad \frac{dI_{out}}{dT} = \frac{1}{R} \frac{dV_{T1}}{dT} - \frac{1}{R^2} \frac{dR}{dT}$

In the resistor is polysilicon, then

$TC_F = I_{out} \frac{dI_{out}}{dT} = \frac{1}{V_{T1}} \frac{dV_{T1}}{dT} - \frac{1}{R} \frac{dR}{dT} = -\alpha\frac{1}{V_{T1}} \frac{dR}{dT} = \frac{-2.3\times10^{-3}}{0.7} - 1.5\times10^{-3} = -4786\text{ppm}/^\circ\text{C}$

Base-emitter referenced source:

The output current was given as, $I_{out} = I_2 = \frac{V_{BE1}}{R}$

The $TC_F = \frac{1}{V_{BE1}} \frac{dV_{BE1}}{dT} - \frac{1}{R} \frac{dR}{dT}$

If $V_{BE1} = 0.6V$ and $R$ is poly, then the $TC_F = \frac{1}{0.6} (-2\times10^{-3}) - 1.5\times10^{-3} = -4833\text{ppm}/^\circ\text{C}$. 

**Technique to Make \( g_m \) Dependent on a Resistor**

Consider the following circuit with all transistors having a \( W/L = 10 \). This is a bootstrapped reference which creates a \( V_{\text{bias}} \) independent of \( V_{DD} \). The two key equations are:

\[
I_3 = I_4 \implies I_1 = I_2
\]

and

\[
V_{GS1} = V_{GS2} + I_2 R
\]

Solving for \( I_2 \) gives:

\[
I_2 = \frac{V_{GS1} - V_{GS2}}{R} = \frac{1}{R} \left( \sqrt{\frac{2I_1}{\beta_1}} - \sqrt{\frac{2I_2}{\beta_2}} \right) = \frac{\sqrt{2I_1}}{R\sqrt{\beta_1}} \left( 1 - \frac{1}{2} \right)
\]

\[
\therefore \quad I_2 = \frac{1}{R\sqrt{2\beta_1}} \implies I_1 = \frac{1}{2\beta_1 R^2} = \frac{1}{2 \cdot 110 \times 10^{-6} \cdot 10 \cdot 25 \times 10^6} = 18.18 \mu A
\]

Now, \( V_{\text{bias}} \) can be written as

\[
V_{\text{bias}} = V_{GS1} - V_{TN} = \frac{1}{\beta_1 R} + V_{TN} = \frac{1}{110 \times 10^{-6} \cdot 10 \cdot 5 \times 10^3} + 0.7 = 0.1818 + 0.7 = 0.8818 V
\]

Any transistor with \( V_{GS} = V_{\text{bias}} \) will have a current flow that is given by \( 1/2\beta R^2 \).

Therefore,

\[
g_m = \sqrt{2I_\beta} = \sqrt{\frac{2\beta}{2\beta R^2}} = \frac{1}{R} \implies g_m = \frac{1}{R}
\]

---

**Summary of Reference Performance**

<table>
<thead>
<tr>
<th>Type of Reference</th>
<th>( S_{V_{\text{REF}}}^{V_{DD}} )</th>
<th>( TC_F )</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSFET-R</td>
<td>(&lt;1)</td>
<td>( &gt;1000\text{ppm/°C} )</td>
<td></td>
</tr>
<tr>
<td>BJT-R</td>
<td>(&lt;&lt;1)</td>
<td>( &gt;1000\text{ppm/°C} )</td>
<td></td>
</tr>
<tr>
<td>Gate-Source Referenced</td>
<td>Good if currents are matched</td>
<td>( &gt;1000\text{ppm/°C} )</td>
<td>Requires start-up circuit</td>
</tr>
<tr>
<td>Base-emitter Referenced</td>
<td>Good if currents are matched</td>
<td>( &gt;1000\text{ppm/°C} )</td>
<td>Requires start-up circuit</td>
</tr>
</tbody>
</table>

- A MOSFET can have zero temperature dependence of \( i_D \) for a certain \( v_{GS} \)
- If one is careful, very good independence of power supply can be achieved
- None of the above references have really good temperature independence

Consider the following example:

A 10 bit ADC has a reference voltage of 1V. The LSB is approximately 0.001V. Therefore, the voltage reference must be stable to within 0.1%. If a 100°C change in temperature is experienced, then the \( TC_F \) must be 0.001%/°C or multiplying by \( 10^4 \) requires a \( TC_F = 10\text{ppm/°C} \).